MATH 211 – Calculus I

Reading Assignments - Part I - Answers

R1 - Due by 10am W 8/27:

- **To Read:** Section 1.2, 1.5
- **Email Subject Line:** Math 211 8/27 Your Name
- **Reading Questions:**

  1. What are the four ways to represent a function?
     ANSWER: with words, with a table of values, with a graph, or with a mathematical formula.

  2. Write an equation for the line that passes through the points (1, 2) and (2, 7).
     ANSWER: The equation of this line is given by either \( y - 2 = 5(x - 1) \) or \( y = 5x - 3 \). (They are just two different ways to write the same equation.)

  3. Name an important property that the trigonometric functions possess that power and exponential functions do not.
     ANSWER: One property that is unique to the trig functions is periodicity, which means their values repeat themselves forever.

R2 - Due by 10am F 8/29:

- **To Read:** Sections 1.3
- **Email Subject Line:** Math 211 8/29 Your Name
- **Reading Questions:**

  1. We know that the graph of \( f(x) = x^2 \) is a parabola that opens upward and has vertex at (0, 0). Write a formula for the function whose graph is this same parabola, but with the vertex shifted down 3 units and to the left 5 units.
     ANSWER: The new function would be \( g(x) = (x + 5)^2 - 3 \).

  2. Describe in words how you would determine the graph of \( f(t) = 7 \cos(3t) + 10 \) from the graph of \( g(t) = \cos(t) \). Use the appropriate terminology introduced in this section of the text.
     ANSWER: Beginning with the graph of \( \cos(t) \), we first compress it horizontally by a factor of 3 (so that, instead of it going through one complete period over intervals of length \( 2\pi \), it goes through one complete period over intervals of length \( \frac{2\pi}{3} \)), then stretch it vertically by a factor of 7 (so that, instead of the range being \([−1, 1]\), it becomes \([−7, 7]\)), then (finally) shifting the resulting graph up by 10 units.
R3 - Due by 10am M 9/1:

- **To Read:** Sections 1.6
- **Email Subject Line:** Math 211 9/1 Your Name
- **Reading Questions:**

  1. Explain the “Horizontal Line Test”.
     
     **ANSWER:** This is a test used on the graph of a function to see if that function is one-to-one. If it is possible to draw a horizontal line through the graph so that it intersects the graph at more than one point, the function is not one-to-one. If this is not possible (i.e. if all possible horizontal lines through the graph intersect the graph at most once), then the function is one-to-one.

  2. Explain what the term “one-to-one” function. Give an example of a one-to-one function, and an example of a function that is not one-to-one.
     
     **ANSWER:** A one-to-one function is a function for which no two values of the independent variable have the same value for the dependent variable. The function \( f(t) = e^t \) is an example of a one-to-one function, because no two t-values have the same \( f(t) \)-value. The function \( g(t) = t^2 \) is an example of a function that is NOT one-to-one, since the two values \( t = -2 \) and \( t = 2 \) both have the same \( g(t) \)-value, namely \( g(-2) = g(2) = 4 \).

  3. The text explains how to find the inverse of a one-to-one function given by a formula and by a graph. How do you think it would be done for a formula given by a table?
     
     **ANSWER:** Since the inverse of a one-to-one function is simply the function you obtain by swapping the roles of the independent and dependent variables, an easy way to obtain the inverse of a tabular one-to-one function is to swap the two lists of values.

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R4 - Due by 10am F 9/5:

- **To Read:** Sections 2.1, 2.2
- **Email Subject Line:** Math 211 9/5 Your Name
- **Reading Questions:**

  1. In geometry, a tangent line is defined at a point \( P \) on a circle, and is described as “the line that touches the circle only at the point \( P \).” For most functions (such as the one in Figure 1(b) on page 95 of the text), however, this definition doesn’t work, because the tangent line touches the graph at more than one point. Using words, give a better definition of a “tangent line to the graph of a function \( f \) at a point \( P \).”
     
     **ANSWER:** A tangent line to the graph of a function \( f \) at a point \( P \) is a line that intersects \( f \) at the point \( P \), and which has the same steepness (or “direction”) as \( f \) at that point.

  2. Explain the difference between average velocity and instantaneous velocity.
     
     **ANSWER:** The average velocity is the velocity over a time period, computed by dividing the distance covered by the time elapsed. The instantaneous velocity is the velocity at a given instant of time, much like a speedometer in a car.

  3. Translate into words what the expression \( \lim_{x \to 1} f(x) = 6 \) means. Do not use the word “limit” in your explanation.
     
     **ANSWER:** This means that, if we substitute values of \( x \) that are close to 1 into the function \( f(x) \), the resulting function values are close to 6. \( x \)-values closer to 1 give function values closer to 6.
R5 - Due by 10am M 9/8:

- **To Read:** Sections 2.2
- **Email Subject Line:** Math 211 9/8 Your Name
- **Reading Questions:**
  1. Briefly describe how you can estimate the value of \( \lim_{x \to -1} f(x) \) for a function given by a formula.
     
     **ANSWER:** You can estimate this limit by evaluating \( f(x) \) for a value of \( x \) that is very close (but not equal) to \(-1\). The closer to \(-1\) the chosen point, the better the estimate, in general.
  2. Use the method you have described to estimate the limit (correct to two decimal places) for the function \( f(x) = \frac{(x^2 - 2x - 3) \sin(x)}{x^2 + 3x + 2} \). (Just give your final estimate in your answer.) Explain how you know it is correct to two decimal places.
     
     **ANSWER:** To two decimal places, \( \lim_{x \to -1} f(x) = 3.37 \). I (think I) know this is correct to two decimal places because, when I choose \( x \)-values successively closer to \( x = -1 \) (from both sides), the corresponding values of \( f(x) \) stop changing in the third decimal place, which means it is correct to two places.

R6 - Due by 10am W 9/10:

- **To Read:** Sections 2.3
- **Email Subject Line:** Math 211 9/10 Your Name
- **Reading Questions:**
  1. What is the purpose of having “Limit Laws”?
     
     **ANSWER:** Limit Laws help us to evaluate a limit of a complicated function by breaking the function up into simpler components. In addition, when the function is given by a formula, these Laws help us to determine the exact value of the limit, rather than relying on “educated guesses”.
  2. What is the purpose of having the Squeeze Theorem?
     
     **ANSWER:** In certain cases, the Squeeze Theorem allows us to determine the value of a limit indirectly when the limit is too difficult to determine directly.
R7 - Due by 10am F 9/12:

- **To Read:** Sections 2.4
- **Email Subject Line:** Math 211 9/12 Your Name
- **Reading Questions:**
  1. For a function \( f \) given by a graph, how can we tell visually if \( f \) is continuous at a point \( P \) on the graph?
     **ANSWER:** Visually, if a function \( f \) is continuous at a point \( P \), then a portion of its graph that passes through \( P \) can be drawn without lifting pencil from paper.
  2. Give an example of a function (given by a formula) that is not continuous at \( x = 2 \). Explain how you know your function fits this question.
     **ANSWER:** The function \( f(x) = \frac{1}{x - 2} \) is not continuous at \( x = 2 \), because the function is not defined at that point.
  3. When evaluating \( \lim_{{x \to c}} f(x) \), why is it helpful to know if the function \( f \) is continuous at \( x = c \)?
     **ANSWER:** The definition of continuity states that if \( f \) is continuous at \( x = c \), then \( \lim_{{x \to c}} f(x) = f(c) \). This means that, to evaluate the limit, we need only compute the function value at \( x = c \).

R8 - Due by 10am W 9/17:

- **To Read:** Sections 2.5
- **Email Subject Line:** Math 211 9/17 Your Name
- **Reading Questions:**
  1. Explain in words what the mathematical statement \( \lim_{{x \to 2}} f(x) = \infty \) means. Do not use the word “limit” in your explanation.
     **ANSWER:** This expression means that the value of \( f(x) \) grows larger and larger without bound as the value of \( x \) gets closer and closer to 2, on either side of 2.
  2. Repeat the previous question for the mathematical statement \( \lim_{{x \to \infty}} f(x) = 5 \).
     **ANSWER:** This expression means that the value of \( f(x) \) gets closer and closer to 5 as the value of \( x \) grows larger and larger without bound.
R9 - Due by 10am W 10/1:

- **To Read:** Sections 2.6

- **Email Subject Line:** Math 211 10/1 Your Name

- **Reading Questions:**

  1. Geometrically, what does the expression $\frac{f(a+h) - f(a)}{h}$, which appears in the definition of the derivative in this section, represent?

     **ANSWER:** This expression represents the slope of a secant line that is drawn between two points on the graph of $f$. The coordinates of the points are $(a, f(a))$ and $(a+h, f(a+h))$. This slope is an approximation to the slope of the tangent line to $f$ at the point $(a, f(a))$.

  2. Do formulas 1 (page 142 in text) and 2 (page 144 in text) represent the same quantity? If so, identify the quantity. If not, briefly explain how the two quantities are different.

     **ANSWER:** Yes, they both represent the slope of the tangent line. They look different only because the coordinates for the secant line points are written differently.

  3. Is it possible to compute the exact value of the slope of the tangent line to a function at a point, if the function is given as a graph or a table? Why or why not?

     **ANSWER:** No, it is not possible, unless the function is linear, in which case the slope of the tangent line is the same as the slope of the linear function itself. The issue is this: To compute the tangent line slope exactly, we must evaluate a limit. When our function is given by a table or graph, there is not enough exact information about function values to evaluate the limit exactly. The best we can do is estimate it.