Evaluating Limits for Functions Given by Formula

Basic Three

- **Exponential Fn.:** $f(x) = a^x$, $a > 0$ constant.
  \[
  \lim_{x \to c} f(x) = f(c) \quad (i.e. \lim_{x \to c} a^x = a^c)
  \]

- **Trig Fn.:** $f(x) = \sin(x)$ or $f(x) = \cos(x)$.
  \[
  \lim_{x \to c} f(x) = f(c) \quad (i.e. \lim_{x \to c} \sin(x) = \sin(c), etc.)
  \]

- **Power Fn.:** $f(x) = x^p$, $p$ constant.
  \[
  \lim_{x \to c} f(x) = f(c), \text{ provided } \text{Dom}(f) \text{ contains an open interval which includes } c.
  \]

Functions Built from Basic Three

- **Arithmetic Combinations:** Assume $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist.
  \[
  \lim_{x \to c} (f(x) + g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) \quad (\text{See Limit Laws 1-5 in text})
  \]

- **Absolute Values:** If $\lim_{x \to c} f(x) = L$, then
  \[
  \lim_{x \to c} |f(x)| = |L|
  \]

- **Inverse Functions:** Assume $f(x)$ is one-to-one.
  - General Case: Later.
  - Logarithms: $f(x) = \log_a(x)$, $a > 0$ constant.
    \[
    \lim_{x \to c} f(x) = f(c) \quad (i.e. \lim_{x \to c} \log_a(x) = \log_a(c))
    \]
  - Inverse Trig Fns.: $f(x) = \sin^{-1}(x)$ or $f(x) = \cos^{-1}(x)$
    \[
    \lim_{x \to c} f(x) = f(c) \quad (i.e. \lim_{x \to c} \sin^{-1}(x) = \sin^{-1}(c), etc.)
    \]

- **Function Composition:** $h(x) = f(g(x))$. If $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} f(x) = M$ both exist, then
  \[
  \lim_{x \to c} h(x) = M
  \]

- **Piecewise Functions:** Handled on case-by-case basis. (Pay attention to the case when $c$ is one of the “break points” of the function.)