1. Write an algorithm to search an unordered array of integers for a key.

Algorithm.
My solution is shown in Algorithm 1. Essentially, the algorithm begins by checking the first element of the array. If the element matches the key, the algorithm returns true. Otherwise, the algorithm checks the next index. This process is repeated until the algorithm is found or all of the elements have been checked.

Algorithm 1 Linear Search
Input: Array A, key k
Output: Boolean value stating existence of k in A

1: for all $a \in A$
2: if $a = k$
3: return true
4: return false

Time Complexity.

Theorem 1. Linear Search (Algorithm 1) runs in $O(n)$ time, where $n$ is the number of elements of the input array.

Proof. In the worst case, Algorithm 1 must visit each element of the array, either finding the key at the final index of the array or not finding the element at all. Thus, the for-loop of the algorithm runs at most $n$ times. Inside of the loop, comparing numbers and looking up an element of an array both take $O(1)$ time. In total, the algorithm runs in $O(n)$ time.

Memory Complexity.

Theorem 2. Linear Search (Algorithm 1) runs in $O(1)$ memory.

Proof. Algorithm 1 will not define any additional memory to the input array. The array will be stored as a pointer value and the for loop needs at most one integer to track the current element’s index. This totals to $O(1)$ extra memory usage.