PERFORMANCE

EFFICIENCY

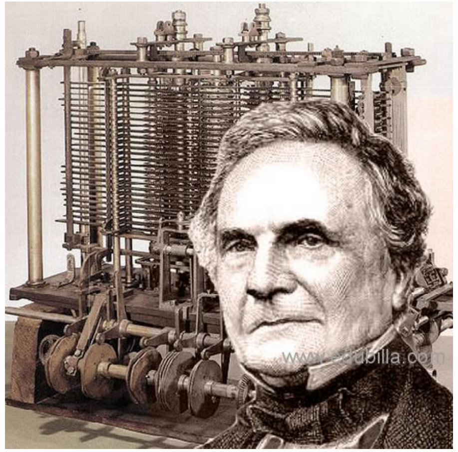
SEARCHING

SORTING

WHAT IS PERFORMANCE?

- Since the early days in computing, computer scientists have concerned themselves with improving hardware, software, visualizations, etc
- Performance can mean many different things

- "The economy of human time is the next advantage of machinery in manufactures." – Charles Babbage



EXAMPLES OF PERFORMANCE

- Fewest computations
- Smaller memory usage
- Faster computations
- Improving accuracy of computations

- How we achieve these
  - Better algorithms
  - Better hardware
  - Better languages

WHY DO WE CARE?

- We want to solve real problems (large) in real time

BIG-OH COMPLEXITY

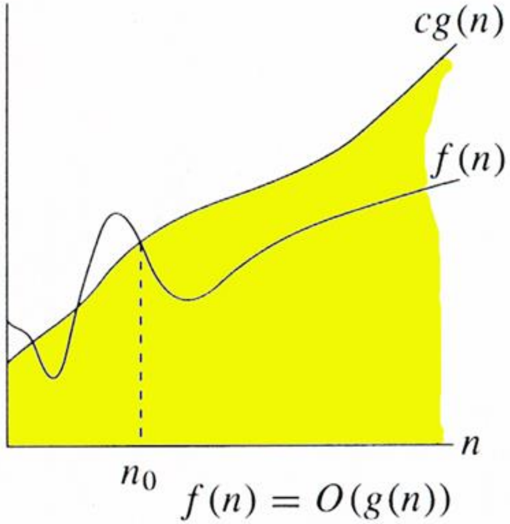
- We will focus our study of performance on time as a metric of performance
- We can measure time experimentally like a stopwatch in our programs:

```
long start = System.nanoTime();
//run algorithm
long stop = System.nanoTime();
double time = (stop - start)/1e9;
```

- We can measure time theoretically with big-oh analysis – an approximation technique for quantifying the time an algorithm takes

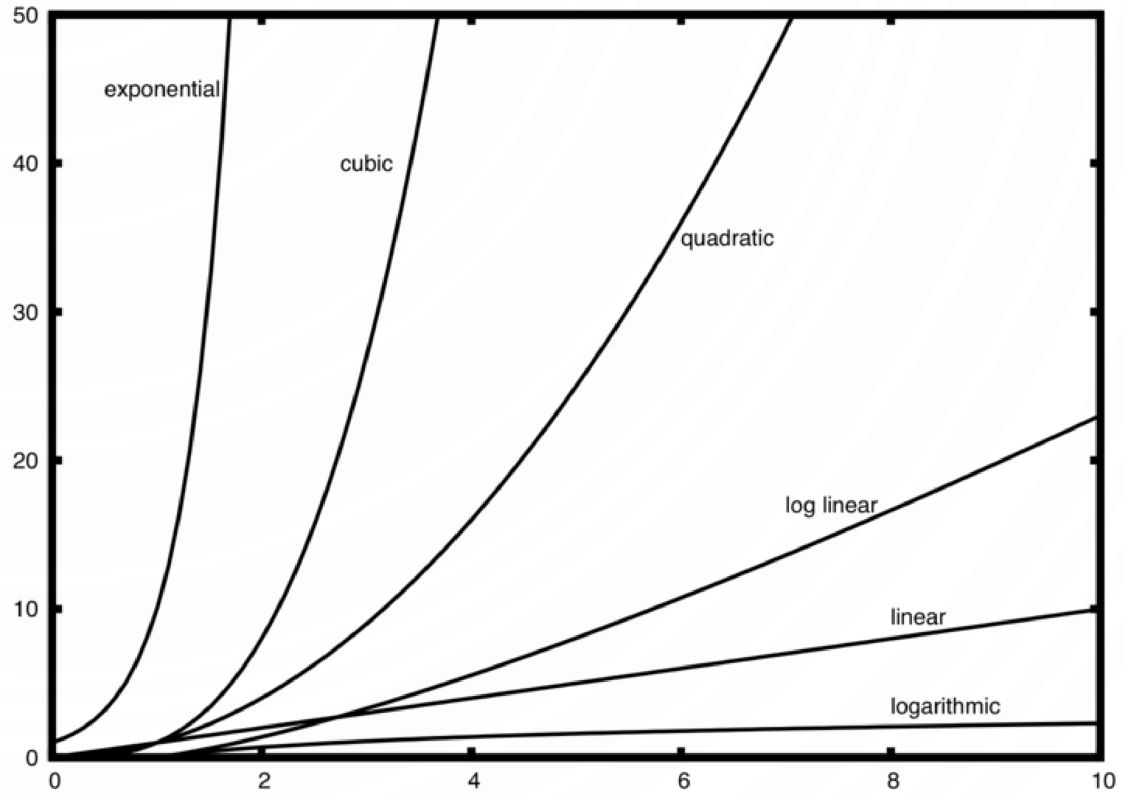
BIG-OH COMPLEXITY

- A function $f(n)$ is $O(g(n))$ (pronounced "big-oh") if there exists constants c, and n_0 such that $f(n) \leq cg(n)$ for all $n \geq n_0$
  - $f(n)$ – real time taken for an algorithm. This is what we want to approximate
  - $g(n)$ – a function that "approximates" $f(n)$, more precisely it is an upper bound to $f(n)$
- We use this, as it describes how long an algorithm will take to compute as the problem size (n) increases
- To determine – count the operations



COMMON BIG-OH FUNCTIONS

- Logarithmic – $O(\log n)$
- Linear – $O(n)$
  - Example: searching for the minimum in an array. We must "look at" all n elements of an array
- Linearithmic – $O(n \log n)$
- Quadratic – $O(n^2)$



SHAMELESS PLUG FOR CMSC 221

- This class is not about how we come up with these equations, or how we design better algorithms. For continued information, continue on in CS coursework

- In this class, I want you to have an intuitive feel of what big-oh means through a few algorithms
- In this class, understand the algorithms I present, but I do not expect you to come up with it yourself

LETS EXPLORE THESE CONCEPTS

- Case study on Searching
  - Linear Search
  - Binary Search

- Case study on Sorting
  - Bubble Sort
  - Selection Sort
  - Merge Sort

WAIT…HOW DO WE DO EXPERIMENTS?

- We vary the size of the data (usually by powers of two), so test on

$$n = 2^1, 2^2, \ldots, 2^d$$

- Repeat each experiment numerous times to:
  - Get an accurate time for operations faster than 1 microsecond (usually one tick of the clock)
  - Average timing considering other tasks running on the computer

- Pseudocode

for $N \leftarrow 2^1 \ldots 2^d$ **do**
    Setup before timing
    $start \leftarrow$ time()
    for $k \leftarrow 0 \ldots repeats$ **do**
        experiment()
    $stop \leftarrow$ time()
    output($\frac{start-stop}{repeats}$)

CASE STUDY OF SEARCHING

LINEAR SEARCH

- Pseudocode

```
Input: Array arr, Key k
Output: true if arr contains k, false otherwise
1. for each a ∈ arr do
2.    if a = k then
3.       return true
4. return false
```

- Complexity?
  - Linear – $O(n)$
  - Reasoning – The search might have to visit each of the n elements contained in the array.
  - Note – it doesn't matter if the first element is equal to the key, that is a *special case*. On average we must search $\frac{n}{2}$ elements. Additionally, we don't care about a specific size, we are interested in performance as the size tends to infinity

CAN WE DO BETTER?

- Computer scientists always ask this kind of question, *can we do better?*
- Well in general…no, this is about the best we can do with searching.
- Computer scientists then ask a follow-up questions, *can we do better in special cases?*
- Yes! If we knew the input was sorted we could do much better.

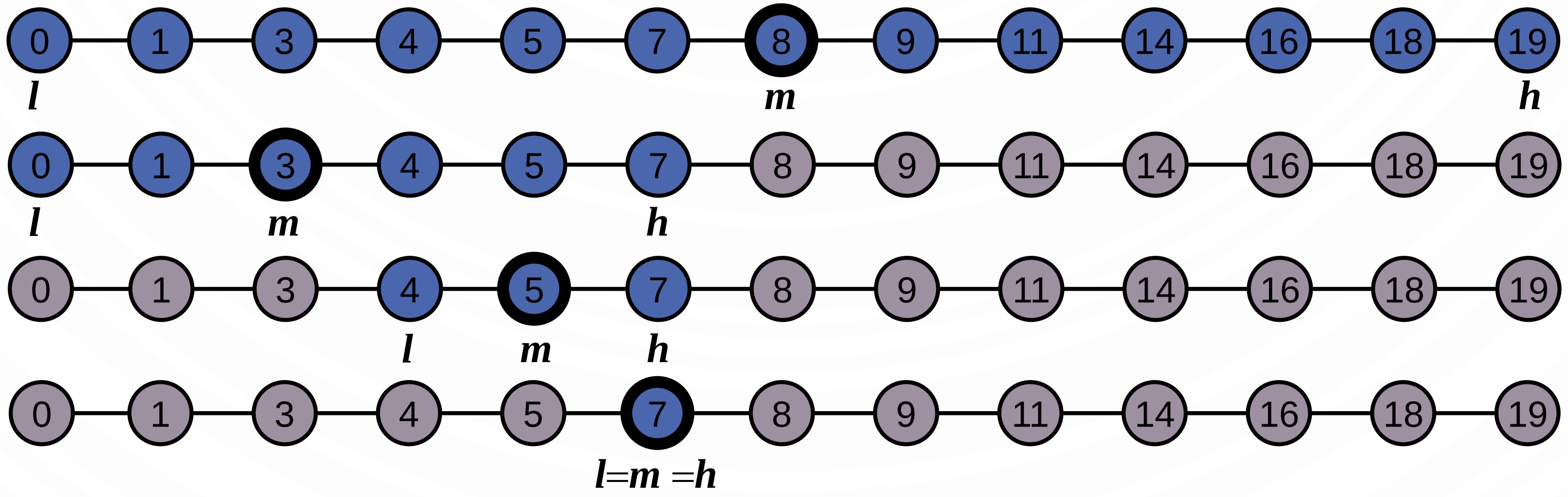
BINARY SEARCH

- Pseudocode

```
Input: Sorted array arr, Key k
Output: true if arr contains k, false otherwise
low ← 0
high ← arr.length − 1
while lo ≤ hi do
    mid ← (high+low)/2
    if k < arr[mid] then
        high ← mid − 1
    else if k > arr[mid] then
        low ← mid + 1
    else
        return true
return false
```

BINARY SEARCH

- How it works?
- Key is 7

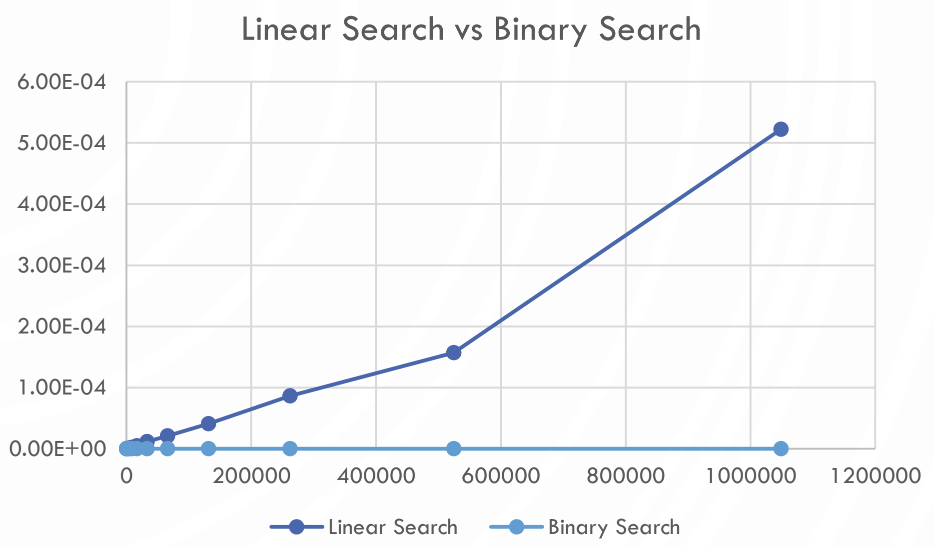


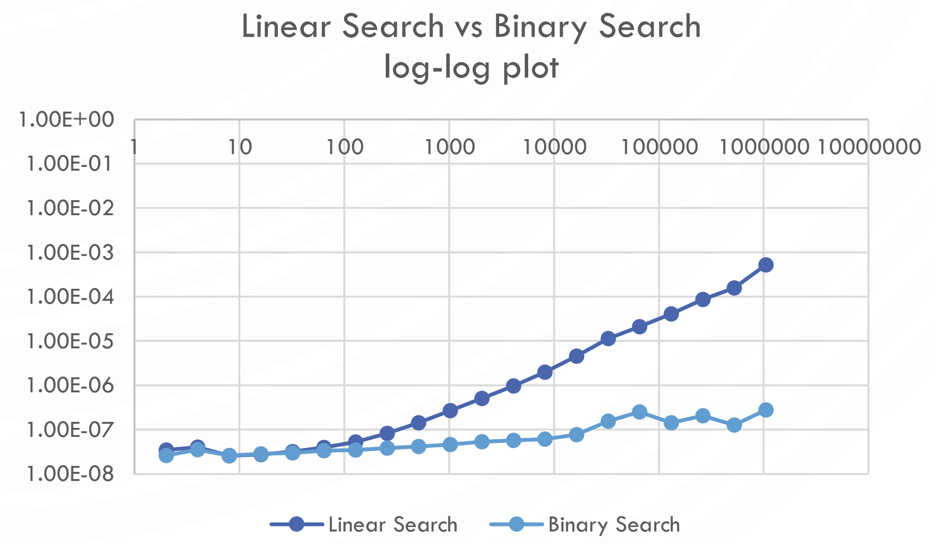
BINARY SEARCH

- Complexity?
  - Logarithmic – $O(\log n)$
  - Reasoning – in each iteration of the loop, we eliminate half of the indices as possible cells to hold the key. The number of times you can repeatedly divide a number by 2 is the definition of a logarithm
  - Note – I am loose on the base of the logarithm. If you feel more comfortable with one, it will always be base 2. However, in big-oh complexity the base doesn't matter. See me after class if you would like a proof.

EXPERIMENT SEARCHING

- Download Search.java from the course website. It contains an experiment ready to go comparing the different searches. Lets go through the file to ensure we understand each component.
- Run the file, open up the csv file in Microsoft Excel
- Make a line scatter plot of the size vs the time of the methods
    - Convert to a log-log plot to get a better picture of the data





CONCLUSION

- A smaller complexity drastically affects runtime
- $O(\log n)$ is much faster than $O(n)$

CASE STUDY OF SORTING

BUBBLE SORT

- Pseudocode

```
Input: Array arr
Output: Sorted array
for i ← 1 ... arr.length do
    for j ← 0 ... arr.length − i do
        if arr[j] > arr[j+1] then
            swap(arr, j, j+1)
```

- Complexity
  - Quadratic – $O(n^2)$
  - Reasoning – There are n passes over the array, in each pass n elements are visited and possibly swapped. $n * n = n^2$

6 5 3 1 8 7 2 4

CAN WE DO BETTER?

- Computer scientists always ask this kind of question, *can we do better?*
- Identify the weakness here, bubble sort swaps too much
- Can we fix it?

SELECTION SORT

- Pseudocode

```
Input: Array arr
Output: Sorted array
for i ← 0 … arr.length − 2 do
    min ← i;
    for j ← i … arr.length − 1 do
        if arr[j] < arr[min] then
            min ← j
    swap(arr, i, min)
```

- Complexity?
  - Quadratic – $O(n^2)$
  - Reasoning – In each iteration of the outer loop, we must find the minimum in the rest of the array, and we swap this minimum into place. Doing this n times, takes in total $O(n^2)$ operations.

	8
	5
	2
	6
	9
	3
	1
	4
	0
	7

CAN WE DO BETTER?

- This was not satisfying, bubble sort and selection sort have the same complexity, even though selection sort is a much nicer idea (and performs better in practice, will see soon)
- Computer scientists always ask this kind of question, *can we do better?*
- Maybe we can try a radically different idea

MERGE SORT

- Split the array in half
- Sort each half recursively
- Merge the two back together

6 5 3 1 8 7 2 4

MERGE SORT

- Pseudocode Sort

Input: Array arr
Output: Sorted array

```
if arr.length < 2 then return
l, r ← split(arr)
MergeSort(l)
MergeSort(r)
arr ← merge(l, r)
```

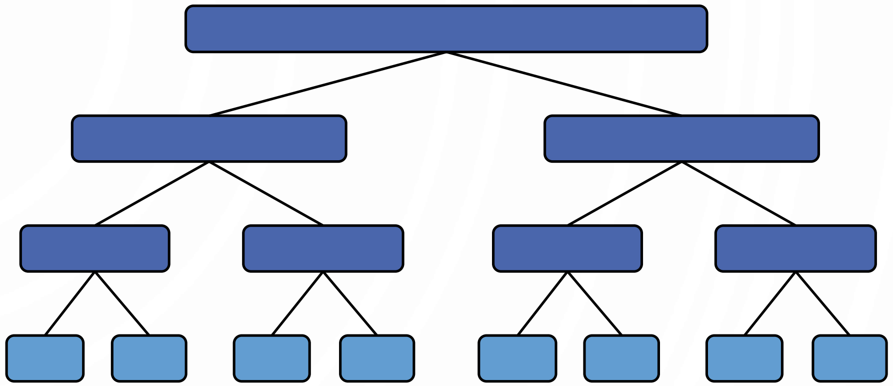
- Pseudocode Merge

Input: Sorted arrays l and r
Output: Sorted array arr

```
arr ← newArray(l.length, r.length)
i ← 0; j ← 0; k ← 0
while i < l.length ∧ j < r.length do
  if l[i] < r[j] then
    arr[k] ← l[i]; k ← k + 1; i ← i + 1
  else
    arr[k] ← l[j]; k ← k + 1; j ← j + 1
while i < l.length do
  arr[k] ← l[i]; k ← k + 1; i ← i + 1
while j < r.length do
  arr[k] ← l[j]; k ← k + 1; j ← j + 1
return arr
```

MERGE SORT

- Complexity?
  - Linearithmic – $O(n \log n)$
  - Reasoning – At each iteration of the recursive function we split the array in half and merge it back together. This is n work. Then we do this same amount of work at each level of the recursion tree. Since we split in half repeatedly, there are a logarithmic number of levels. Thus – n work on $\log n$ levels is $O(n \log n)$



depth	#seqs	size	Cost for level
0	1	n	n
1	2	n/2	n
…	…	…	
i	2^i	$\frac{n}{2^i}$	n
…	…	…	
$\log n$	$2^{\log n} = n$	$\frac{n}{2^{\log n}} = 1$	n

EXPERIMENT SORTING

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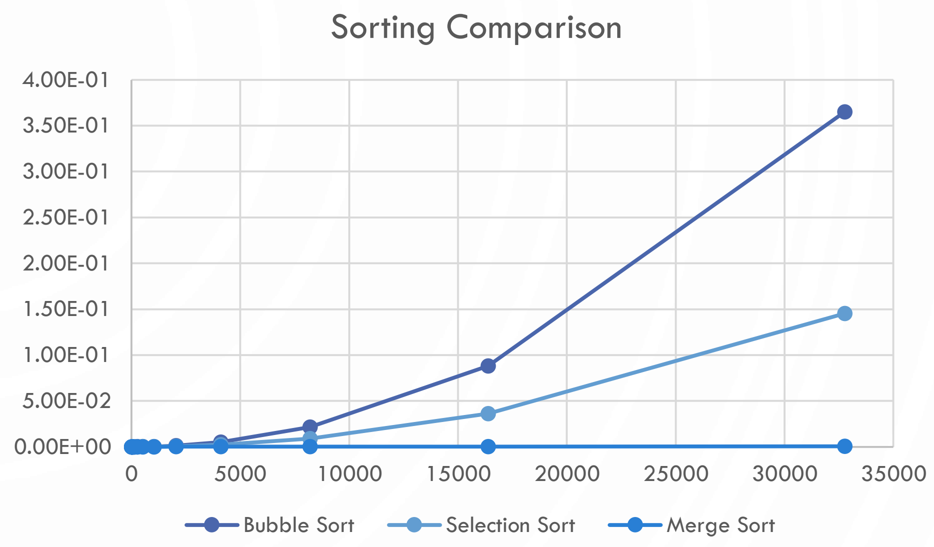

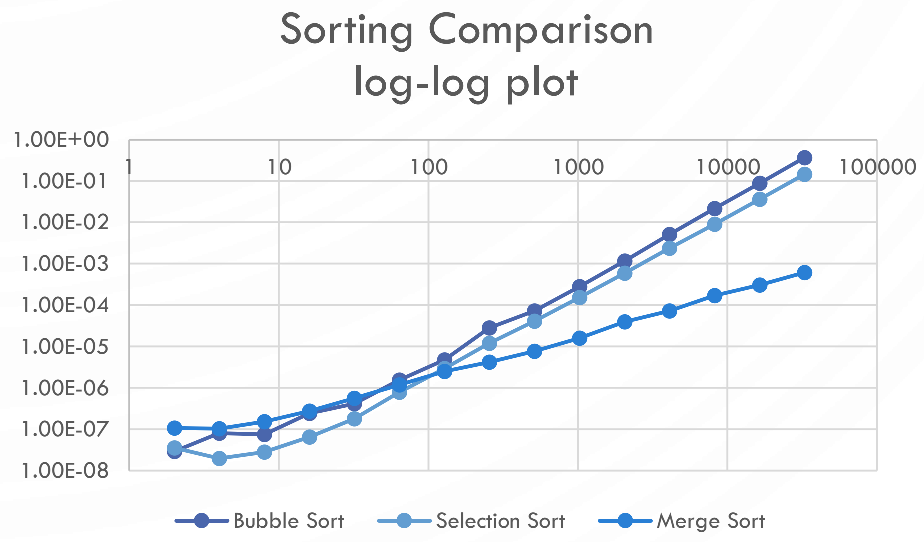

CONCLUSION

- Two algorithms can have the same complexity, but different actual performance
  - We need to experiment on our data
- Smaller complexity will always beat an optimized higher complexity
  - However, note that this doesn't necessarily apply to small values of n
  - Lesson – choosing an appropriate algorithm requires understanding the size of your data

ALGORITHM SUMMARY

- Searching
  - Linear Search – linear time or $O(n)$
  - Binary Search – logarithmic time or $O(\log n)$
- Sorting
  - Bubble Sort – quadratic time or $O(n^2)$
  - Selection Sort – quadratic time or $O(n^2)$
  - Merge Sort – linearithmic time or $O(n \log n)$