

TEST 2

Davis
M211

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(20pts.)

1. a. State the definition of the derivative function $f'(x)$, and use that definition to figure out the derivative function for $f(x) = x^3 - 2x^2$ (show all work, including the calculation of the limit).
- b. Compute the derivative of $y = 2x^7 - \pi x^4 + 13$ and $y = \sqrt{3}(\frac{1}{2})^x$ (you may do these the “quick” way).

The derivative function is defined by $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if that limit exists.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - 2(x+h)^2 - (x^3 - 2x^2)}{h} = \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x^2 - 4xh - 2h^2 - x^3 + 2x^2}{h} = \\ &= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2 - 4x - 2h)}{h} = \\ &= 3x^2 - 4x \end{aligned}$$

$$\frac{dy}{dx} = 14x^6 - 4\pi x^3; \frac{dy}{dx} = \sqrt{3} \ln 1/2 (\frac{1}{2})^x$$

(20pts.)

2. A delivery pizza company's revenue is a function of advertising expenditures, both measured in hundreds of dollars. If we write this relationship as $R = f(a)$, where R is revenue and a is advertising, explain what $f(5) = 30$ means. Explain what $f'(5) = 2$ means. Finally, if $f''(5) < 0$, how does that help you make a decision about how much to increase your advertising spending?

$f(5) = 30$ means that the company will make \$3000 if they spend \$500 on advertising.

$f'(5) = 2$ means that the next \$100 spent on advertising will result in an increase in revenue of \$200.

$f''(5) < 0$ means that the rate of increase of revenue related to spending on advertising is decreasing. Thus, you are getting "less bang for the buck". When the first derivative falls below 1, then it costs a dollar for advertising for less than a dollar in revenue, and that doesn't make any sense (or cents!).

(20pts.)

3. The proportion of students who score between 1100 and 1300 on the SATs is equal to the area under the curve $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ from $x = 1$ to $x = 3$.
 - a. How many subintervals will you need to estimate this area within .1?
 - b. Use the number of subintervals you calculated in part a. to compute the left hand sum to approximate the area under the curve. What percent of the population scores between 1100 and 1300 on the SATs? Is your answer an overestimate or an underestimate?

$|f(b) - f(a)|\delta x \leq .1$ is the equation that is needed to figure out the number of subintervals. In this case, $\delta x = \frac{2}{n}$, $f(3) = .0044318$, $f(1) = .2419707$, so $n \geq 4.75$. Thus, use 5 subintervals.

$$LHS = (2/5)\left(\frac{1}{\sqrt{2\pi}}e^{-1/2} + \frac{1}{\sqrt{2\pi}}e^{-(1.4)^2/2} + \frac{1}{\sqrt{2\pi}}e^{-(1.8)^2/2} + \frac{1}{\sqrt{2\pi}}e^{-(2.2)^2/2} + \frac{1}{\sqrt{2\pi}}e^{-(2.6)^2/2}\right) \cong .2$$

About 20% of the people taking SATs score in the range from 1100 to 1300. This is an overestimate because all of the rectangles lie above the curve because the function is decreasing.

(20pts.)

4. Sketch a picture of a function $y = f(x)$ on the interval from $x = 1$ to $x = 1 + h$ for $h > 0$ so that the function is increasing and concave down on that interval (there are a lot of pictures that will work for this!).

A. Each of the following quantities is either a slope, a length, or an area. Identify which is which, and then show on your picture what these quantities represent.

- a. $f(1 + h) - f(1)$ This is a length in the vertical direction.
- b. $f'(1 + h)$ This is the slope of a tangentline at $1 + h$.
- c. $f(1)h$ This is the area of the rectangle of height $f(1)$ and width h .

B. Put the following in order from biggest to smallest: $\frac{f(1+h)-f(1)}{h}$, $f'(1)$, and $f'(1+h)$. Give a brief explanation of your ordering. Because of the concavity, the biggest is $f'(1)$, then $\frac{f(1+h)-f(1)}{h}$, and the smallest is $f'(1+h)$.

(20pts.)

5. When you bungee jump, you fall at approximately $v = 9.8t$ meters per second. State the fundamental theorem of calculus and use it to determine how far you will fall in 3 seconds (you may do this graphically or algebraically).

The Fundamental Theorem of Calculus states that if $F'(x) = f(x)$ in the range $a \leq x \leq b$, then $\int_a^b f(x)dx = F(b) - F(a)$. In this case, $f(t) = 9.8t$, and $F(t) = 4.9t^2$. The distance you fall in 3 seconds is $F(3) - F(0) = 4.9(9) = 44.1$ meters. You certainly hope the bungee operators are able to do basic calculus to figure out how far you are going to fall!