

### The game of SETS

The goal of the game of SETS is to collect as many SETS of 3 cards as possible. Each card has four descriptive factors: (i) the number of objects (1,2, or 3); (ii) the color of the objects (pink, blue, or black); (iii) the shape of the objects (circle, triangle, or square); and (iv) the shading of the objects (open, striped, or solid). A “SET” of these cards is a collection of 3 cards so that each factor is either the same on each card or is different on each card. For example, if we use the notation (number, color, shape, shading) to describe each card, the following is a SET: (1,blue,circle,striped); (3,pink,circle,striped); and (2,black,circle,striped).

- 1 Play the game in your group for about 10 minutes. To play, place about 12 cards on the table and everyone look for SETS. When someone claims to have a SET, everyone else should verify the collection. If everyone agrees, then those cards are collected by the person who identified them, and 3 new cards are placed on the table. Continue this until all the cards are used (or until 10 minutes is up!).
- 2 How many cards are there in the deck?
- 3 If you represent each factor by a number, find a way to use arithmetic to verify that a collection of 3 cards is a SET. (You might choose pink as 1, blue as 2, and black as 3, etc.)
- 4 (Easy question): Choose a pair of cards at random out of the deck: how many cards can be used to complete a SET for those two cards? Use your answer to 3 to justify your answer.
- 5 (Medium question): Choose one card at random out of the deck: how many pairs of cards can be used to complete a SET for that card? Use your answer to 3 to justify your answer.
- 6 (Hard question): What is the maximum number of cards you can place on the table so that no collection of 3 cards forms a SET?
- 7 (Coding theory question): Define a code to be the set of 12-tuples, each component consisting of a 1,2, or 3, so that the 3 constituent 4-tuples form a SET (the example in the introduction could be the 12-tuple (1,2,1,2,2,1,1,2,3,3,1,2)). How many legitimate 12-tuples are there? What types of mistakes can be detected? What types of mistakes will not be detected?