TEST 3

Davis — Name:

M235 — Pledge:

Show all work; unjustified answers may receive less than full credit.

(17pts.) **1.** Compute $\int_{\mathbf{C}}(8x+36xy)ds$, where $c(t)=(t,t^2,t^3)$ on the interval $0\le t\le 1$.

$\int_{\mathbf{C}}(8x+36xy)ds=\int_0^1(8t+36t^3)\sqrt{1+4t^2+9t^4}dt=\frac{2}{3}(1+4t^2+9t^4)^{\frac{3}{2}}|_0^1=\frac{2}{3}((14)^{\frac{3}{2}}-1)$.

(17pts.) **2.** Compute $\int_{\mathbf{C}}F\cdot ds$, where $F(x,y,z)=(3x^2y^2z,2x^3yz,x^3y^2)$ and C is a curve from $(3,2,1)$ to $(1,2,3)$ (hint: there is an easy way to do this problem).

If you recognize that $F=\nabla f$ for $f(x,y,z)=x^3y^2z$, then $\int_{\mathbf{C}}F\cdot ds=f(1,2,3)-f(3,2,1)=12-108=-96$.

(16pts.) **3.** Evaluate $\int\int_{\mathbf{S}}(x+y+z)dS$ across the rectangle with vertices $(1,1,1),(2,3,4),(-1,2,1)$, and $(0,4,4)$.

Parametrize the surface by $(1,1,1)+u(1,2,3)+v(-2,1,0), 0\le u\le 1, 0\le v\le 1$. In this case, the length of $T_u\times T_v$ is $\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -2 & 1 & 0\end{vmatrix}$, which is $\sqrt{70}$. Thus, $\int\int_{\mathbf{S}}(x+y+z)dS=\sqrt{70}\int_0^1\int_0^1(3+6u-v)dudv=\frac{11}{2}\sqrt{70}$.

(17pts.) **4.** Let the velocity field of a fluid be described by $F=xi+yj$ (measured in meters per second). Compute how many cubic meters of fluid per second are crossing the surface of $z=4-x^2-y^2, z\ge 0$, in the direction of increasing z.

$\int\int_S(x,y,0)\cdot(2x,2y,1)dxdy=\int_0^{2\pi}\int_0^2 2r^2rdrd\theta=\int_0^{2\pi}8d\theta=16\pi$.

(16pts.) **5.** Use Green's Theorem to show that the area contained by an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

Parametrize the ellipse by $x=a\cos(\theta)$ and $y=b\sin(\theta)$. Green's Theorem states that $Area=\int\int_D dxdy=\frac{1}{2}\int_C xdy-ydx=\frac{1}{2}\int_0^{2\pi}(a\cos(\theta)b\cos(\theta)-(b\sin(\theta))(-a\sin(\theta)))d\theta=\pi ab$.

(17pts.) **6.** State Stokes' Theorem, and explain why both integrals in Stokes' Theorem will be 0 if the function $F=\nabla f$ for some f (there is a different reason for the two integrals).

Stokes' Theorem states that under suitable conditions on the function and the surface, $\int\int_S(curlF)\cdot dS=\int_{\partial S}F\cdot dS$. If $F=\nabla f$ for some f, then $curlF=\nabla\times\nabla f=0$, so $\int\int_S(curlF)\cdot dS=0$. In the line integral, if $F=\nabla f$ for some f, then $\int_{\partial S}F\cdot dS=f(c(b))-f(c(a))$. Since ∂S is a simple closed curve, $c(b)=c(a)$ and hence $\int_{\partial S}F\cdot dS=0$.