TEST 1

Davis
M212

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(20pts.) **1.** Without your calculators (but possibly using the tables), compute the following:

a. $\int_1^2 xe^{x^2}dx$

$1/2(e^4 - e)$

b. $\int \arcsin(5x)dx$

$x\arcsin(5x) + 1/2(1-25x^2)^{1/2} + C$

c. $\int e^{2t}\cos(3t)dt$

$1/13e^{2t}(3\sin(2t) + 2\cos(3t)) + C$

d. $\int \frac{x+3}{x^2+4x+5}dx$

$1/2\ln(x^2+4x+5) + \arctan(x+2) + C$

(20pts.) **2.** For the function pictured below, suppose we are trying to estimate $\int_0^2 f(x)dx$. Determine whether the following statements are true or false and give a one sentence justification:

a. $Right(n) \leq Trap(n)$ False: Right is overestimate, Trap is underestimate.

b. $Trap(n) \leq Simp(n)$ True: Simp is average of Trap and Mid, and Trap is an underestimate, so Simp is bigger.

c. $Mid(n) \leq Left(n)$ False: Mid is overestimate, Left is underestimate.

d. $\int_0^2 f(x)dx \leq Left(n)$ False: Left is underestimate of exact value.

e. $Right(1000) \leq Right(2000)$ False: $Right(2000)$ is less of an overestimate than $Right(1000)$ (it is more accurate).

(10pts.) **3.** For the function from problem 2, estimate $Left(4)$, state whether it is an overestimate or underestimate, and give a rough estimate of how far it is from the actual answer.

$Left(4) = (1/2)(1 + 1.02 + 1.03 + 1.035) = 2.0225$. This is an underestimate, and the fact that $Right(4) = 2.0425$ we see that it is at worst .02 away from the true answer (Right is an overestimate, Left is an underestimate, and the true answer is in between them).

(12pts.) **4.** Do the following integrals converge or diverge? Explain your answer.

a. $\int_5^\infty \frac{2-\sin(z)}{z^{\frac{2}{3}}}dz$

Diverges because the integral has more area than the area under $z^{-2/3}$ from 1 to ∞, and that curve has an infinite area.

b. $\int_3^5 \frac{1}{(x-3)^{\frac{2}{3}}}dx$

Converges, $3(2)^{1/3}$.

(10pts.) **5.** Find the volume of a cone whose height is 3 cm and whose base radius is 1 cm.

Draw a picture of a slice of the cone with cross-sectional area a circle of radius r at height h from the base. Using similar triangles, you can see that the volume of the slice is $\pi(\frac{3-h}{3})^2\delta h$. Turning this into an integral you get $\int_0^3 \pi(\frac{3-h}{3})^2 dh = \pi$.

(20pts.) **6.** Verify that $\int x^n \ln(x)dx = \frac{1}{n+1}x^{n+1}\ln(x) - \frac{1}{(n+1)^2}x^{n+1} + C, n \neq -1$ using integration by parts.

Let $u = \ln(x)$ and $dV = x^n dx$: then $du = 1/x$ and $V = \frac{x^{n+1}}{n+1}$. Applying parts, we get $\int x^n \ln(x)dx = \ln(x)\frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1}(1/x)dx$. This last part integrates to $\frac{x^{n+1}}{(n+1)^2}$ as required.

(8pts.) **7.** Evaluate exactly $\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{2i}{n})^2\frac{2}{n}$ without using your calculator (hint: interpret the sum as a Right hand sum and rewrite this as an integral).

$\lim_{n\to\infty}\sum_{i=1}^{n}(1+\frac{2i}{n})^2\frac{2}{n} = \int_0^2(1+x)^2dx = \frac{26}{3}$.