Quiz 6

Davis
M212

Name:
Pledge:

(8pts.) 1. Compute the 4^{th} degree Taylor polynomial $P_4(x)$ near $x = 0$ for $f(x) = (1 + 2x)^{-1}$. Use $P_4(x)$ to approximate $f(.25)$, and compare to the actual value of $f(.25)$.

$P_4(x) = 1 - 2x + 4x^2 - 8x^3 + 16x^4$; $P_4(.25) = 1 - 1/2 + 1/4 - 1/8 + 1/16 = .6875$. The actual value is $f(.25) = 2/3 = .6666\ldots$, so the approximation is about .0209 off.

(8pts.) 2. Find the interval of convergence for the Taylor series for $f(x) = (1 + 2x)^{-1}$: determine the convergence for the endpoints of the interval if there are endpoints.

The n^{th} coefficient of the Taylor Series is $(-2)^n$, so the Ratio Test is $R = \lim_{n\to\infty} \frac{(-2)^n}{(-2)^{n+1}} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2}$, implying that the interval of convergence is $|x| < \frac{1}{2}$. For the endpoints, when we plug in $x = \frac{1}{2}$, we get $1 - 1 + 1 - 1 + 1 - \cdots$, which does not converge, and when we plug in $\frac{-1}{2}$, we get $1 + 1 + 1 + \cdots$, which also does not converge, so neither endpoint converges.

(4pts.) 3. We showed in class the the series $S(x) = \frac{x^2}{4} - \frac{x^3}{9} + \frac{x^4}{16} - \frac{x^5}{25} + \frac{x^6}{36} - \cdots$ converges for $|x| < 1$ (this is essentially the negative of homework problem 17). Assuming this to be true, answer the following.

a. Use a right-hand sum of $f(x) = 1/x^2$ with $\Delta x = 1$ on the interval from 1 to ∞ to determine the convergence of $S(x)$ at the endpoint $x = -1$ (state whether the series converges or not, and use the right-hand sum to justify your answer. A picture would really help!).

The picture would indicate that the right hand sum is equal to the series listed above, and it is an underestimate for the integral $\int_1^\infty \frac{1}{x^2} dx$. From our work earlier this semester, this integral has area 1, so the series must converge at $x = -1$.

b. Does $S(x)$ converge for $x = 1$? Explain your answer. The alternating series test can be applied in this case because the terms are all getting smaller and going to 0, implying that the series converges for $x = 1$.