Quiz 6

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M212 — Pledge:

(8pts.) 1. The Pacific halibut fishery has been modeled by the differential equation $\frac{dP}{dt} = .4P(1 - \frac{P}{400}) - 30$. Use Euler's method with step size $h = 5$ to approximate the number of fish you have after 10 weeks starting with $P(0) = 120$.

We start with $y_0 = 120$; the first slope is $\frac{dP}{dt} = .4(120)(1 - \frac{120}{400}) - 30 = 3.6$. With a step size of $h = 5$, we get $y_1 = 3.6(5) + 120 = 138$. Recomputing the slope gives $\frac{dP}{dt} = .4(138)(1 - \frac{138}{400}) - 30 = 6.156$, yielding $y_2 = 6.156(5) + 138 = 168.78$, so we would expect about 169 fish after 10 weeks.

(8pts.) 2. A glucose solution is administered intravenously into the bloodstream at a constant rate. The glucose is converted into other substances at a rate proportional to the concentration at the time. The differential equation describing the concentration $y(t)$ is

$$\frac{dy}{dt} = 2 - y$$

Solve this differential equation. If $y(0) = .4$, find $y(.5)$ (time is being measured in hours, so $t = .5$ is half an hour later).

We need to separate the variables here and integrate. Separating the variables, we get $\frac{dy}{2-y} = dt$, and then integrating yields $-\ln(2-y) = t + C$. Solving for y, we get $\ln(2-y) = -t - C; 2 - y = e^{-t-C}; y = 2 - C'e^{-t}$. Since $y(0) = .4$,, we get $C' = 1.6$, so $y(.5) = 2 - 1.6e^{-.5} = 1.03$.

(4pts.) 3. The half-life of cesium-137 is 30 years (the rate of decay of this radioactive material is proportional to the amount present). Suppose we have a 100 mg sample. How much of the sample remains after 100 years?

Use the half-life to determine the k in the decay equation: $\frac{1}{2}(100) = 100e^{k(30)}; k = \frac{\ln(.5)}{30}$. The amount remaining after 100 years is $100e^{\frac{\ln(.5)}{30}(100)} = 9.92$ mg.