Quiz 3

Davis
M212

Name:
Pledge:

(8pts.) 1. Show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

There were two keys to doing well on this problem. First, writing down the equation $V_{\text{slice}} = \pi R^2 \Delta y$ comes from the observation that all of the slices are circles. The second key comes from getting R in terms of y, where I measured y from the center of the sphere. If we draw the right triangle with vertices at the center of the sphere, at the height y either above or below the center on the y-axis, and the point at height y on the outside of the sphere, then we can apply the Pythagorean Theorem to get $y^2 + R^2 = r^2$, or $R^2 = r^2 - y^2$. Plugging this in to the formula for V_{slice} yields $V_{\text{slice}} = \pi(r^2 - y^2)\Delta y$. Adding these up gives an approximation for the volume of the sphere, $V_{\text{sphere}} \cong \sum_{i=1}^{n} \pi(r^2 - y_i^2)\Delta y$. If we let $\Delta y \to 0$, we turn this summation into the integral $\int_{-r}^{r} \pi(r^2 - y^2)dy = \pi(r^2 y - \frac{1}{3}y^3)|_{-r}^{r} = \frac{4}{3}\pi r^3$ as claimed.

(6pts.) 2. Set up (but do not solve) the integrals required to compute the volume generated by revolving the area under $y = x^2$, above the x-axis, and between $x = 0$ and $x = 2$ about the line $y = -1$ by (a) the disk/washer method and (b) the shell method. Show your work (don't just write down the formula!).

(a) $V_{\text{slice}} = (\pi(x^2+1)^2 - \pi(1)^2)\Delta x$ (the outer radius is $x^2 - (-1)$ and the inner radius is 1). Turning this into a sum yields $V_{\text{object}} \cong \sum_{i=1}^{n} \pi((x_i^2 + 1)^2 - 1)\Delta x$. Letting $\Delta x \to 0$ yields the integral $\int_0^2 \pi((x^2+1)^2 - 1)dx$.

(b) $V_{\text{shell}} = (2\pi r)(h)\Delta y = (2\pi(y+1))(2-\sqrt{y})\Delta y$ since the radius is $y - (-1)$ and the height is the distance between the right point (2) and the left point on the function ($\sqrt{y}$). Turning this into a sum yields $V_{\text{object}} \cong \sum_{i=1}^{n} 2\pi(y_i + 1)(2 - \sqrt{y_i})\Delta y$. Letting $\Delta y \to 0$ yields the integral $\int_0^4 2\pi((y+1))(2-\sqrt{y}))dx$.

(6pts.) 3. My basement flooded during the recent hurricane. The floor of the basement is about 4 feet below ground level, and I had 0.5 feet of water covering a rectangular basement that is 40 feet by 25 feet. Assuming that water has a density of 62.5 pounds per cubic foot, how much work was required to bail the water out of my basement (the pump that usually does this work doesn't work when we don't have electricity :-()?

The volume of a slice of water is $1000\Delta y$ (my basement is a 1000 square foot rectangle, and the Δy is the thickness of the slice). This will weigh $62500\Delta y$ because of the density of water, and it must be moved $4-y$ feet for a work on the slice of $W_{\text{slice}} = \int_0^{.5} 62500(4-y)dy = 117,187.5$ foot-pounds of work (and yes, my back was aching!).