Answers to Quiz 1

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M212

Name:
Pledge:

(8pts.) 1. Use integration by parts to justify the following formula: $\int \cos^n(x)dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)dx$.

Choose $u = \cos^{n-1}(x)$ and $v' = \cos(x)$, which leads to $u' = (n-1)\cos^{n-2}(x)(-\sin(x))$ and $v = \sin(x)$. Applying integration by parts yields

$$\int \cos^n(x)dx = \cos^{n-1}(x)\sin(x) - (-(n-1))\int \cos^{n-2}(x)\sin^2(x)dx$$

Using the trig identity $\sin^2(x) = 1 - \cos^2(x)$ and the integral property that $\int(f(x) - g(x))dx = \int f(x)dx - \int g(x)dx$ implies

$$\int \cos^n(x)dx = \cos^{n-1}(x)\sin(x) + (n-1)\int \cos^{n-2}(x)dx - (n-1)\int \cos^n(x)dx$$

Adding the last term to the left hand side and dividing by n gives the reduction formula.

(12pts.) 2. Integrate the following:

a. $\int \frac{e^x}{e^x+1}dx$

Using u-substitution with $u = e^x + 1$ leads to $\int \frac{du}{u} = \ln|u| + C = \ln|e^x + 1| + C$

b. $\int \ln t\,dt$

Integration by parts with $u = \ln t$ and $v' = 1$, so $u' = \frac{1}{t}$ and $v = t$. Applying the formula leads to the antiderivative $t\ln t - \int t\frac{1}{t}dt = t\ln t - t + C$.

c. $\int \frac{2x+3}{x^2+3x-10}dx$

Either u-substitution with $u = x^2 + 3x - 10$ or partial fractions with a denominator factorization of $(x+5)(x-2)$ will work. For partial fractions, you should end up with the equations $A(x-2) + B(x+5) = 2x+3$. Plugging in 2 yields $7B = 7$, so $B = 1$. Plugging in -5 yields $-7A = -7$, so $A = 1$. Thus, the integral becomes $\int(\frac{1}{x+5} + \frac{1}{x-2})dx = \ln|x+5| + \ln|x-2| + C$.