Final

Davis
M212

Name:
Pledge:

Show all work; unjustified answers may receive less than full credit.

(25pts.) **1.** Compute the following antiderivatives.

a. $\int_0^1 \arctan(t)dt$

b. $\int \frac{1}{x^2+2x}dx$

c. $\int \frac{1}{x^2+2x+2}dx$

d. $\int e^{5x}\cos(3x)dx$

e. $\int \frac{1}{\sin^3(x)}dx$

(25pts.) **2.** A motor boat is traveling through the water and shuts its engines off at time $t = 0$. The velocity at time t is $v(t) = 25 - t^2$ (in $\frac{ft}{s}$) on the interval [0,5] (for t measured in seconds).

a. Estimate the distance traveled during this time interval using a left-hand sum with $n = 5$ subintervals.

b. Is the left hand sum an overestimate or an underestimate? Roughly how far wrong is the estimate for the actual distance traveled?

c. When estimating $\int_0^5 v(t)dt$, place the following in order: RHS(100), RHS(200), MID(100), and TRAP(200).

(20pts.) **3.** State whether the following integrals converge.

a. $\int_1^\infty \frac{1}{x^2+4x+4}dx$

b. $\int_0^\infty (1+\sin(x))e^{-2x}dx$

c. $\int_0^3 \frac{1}{(x-3)}dx$

d. $\int_2^\infty \frac{1}{x(\ln(x))^2}dx$

(20pts.) **4.** An oil company discovered an oil reserve of 75 million barrels. Suppose that for time $t > 0$, in years, the company's extraction plan is a linear declining function of time satisfying $q(t) = 8.5 - .2t$, where $q(t)$ is the rate of extraction of oil in millions of barrels per year at time t. What is the present value of the company's profit over the next 5 years if the oil price is a constant \$23 per barrel, the extraction cost per barrel is \$11, and the market interest rate is 8% per year? How much oil is left after 5 years?

(25pts.) **5.** Let $P(t)$ be the performance level of someone learning a skill as a function of the training time t. The graph of $P(t)$ is called the learning curve. You are told that the rate of change of the performance level is proportional to the difference between the maximal performance level M (a constant) and the current performance level.

a. Write a differential equation satisfied by $P(t)$ (I will give this to you for 10 points if you are having trouble with this problem).

b. Solve the differential equation from part a.

c. If the performance level is $P = 0$ at time $t = 0$ and $P = \frac{M}{2}$ at $t = 2$, what time does that performance level reach $P = \frac{3M}{4}$?

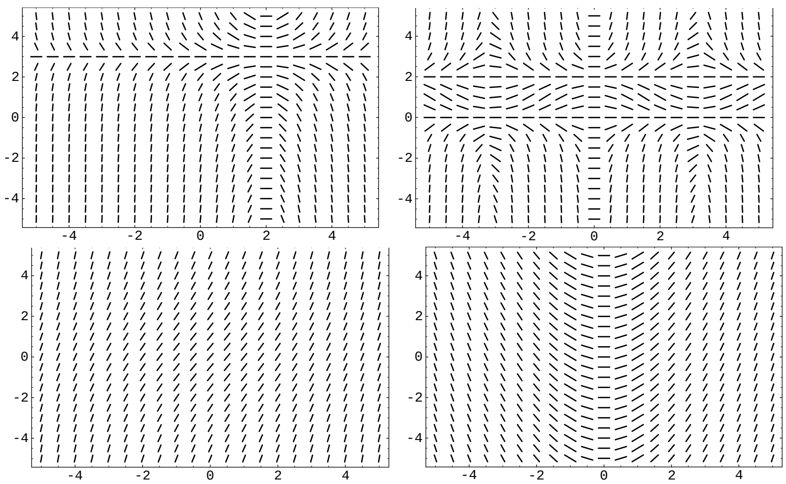
(20pts.) **6.** Match the slope fields below with their differential equations. For each case, identify any equilibrium points and state whether the equilibrium point is stable or unstable.

a. $\frac{dy}{dx} = x$

b. $\frac{dy}{dx} = (x-2)(y-3)$

c. $\frac{dy}{dx} = y(y-2)\sin(x)$

d. $\frac{dy}{dx} = (\frac{x^2}{9}+2)(\frac{y^2}{25}+1)$



(25pts.) **7.** Find the first 3 nonzero terms for the Taylor series about $x = 0$ for the following functions. In the last two parts, estimate the error you would have if you used $P_2(x)$ for an x-value in the interval $[0,.5]$. Find the exact error in one of the two cases (your choice).

a. $\sin(x^2)$

b. $\frac{e^x - 1}{x}$

c. $(1+.5x)^{-\frac{2}{3}}$

d. $\ln(1+.5x)$

(20pts.) **8.** Find the radius of convergence for the following two series. If there are endpoints, determine whether the series converge at the endpoints.

a. $\sum_{n=1}^{\infty} \frac{x^n}{n^2 5^n}$

b. $\sum_{n=1}^{\infty} \frac{x^n}{(2n+1)}$

(20pts.) **9.** True or False (write out the word, but no reason is required).

a. The center of mass of a system is defined as $\frac{\int_a^b x\delta(x)dx}{\int_a^b \delta(x)dx}$.

b. When modeling human population growth, the differential equation $\frac{d^2P}{dt^2} = -kP$ is the differential equation which best describes the population P at time t.

c. The n^th degree Taylor polynomial $P_n(x)$ for $f(x)$ about $x = 0$ satisfies $P_n^{(k)}(0) = f^{(k)}(0)$ for $0 \le k \le n$ (the notation $f^{(k)}(0)$ is the kth derivative evaluated at 0).

d. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

e. If the Taylor series for $f(x)$ about $x = 0$ converges at $x = 2$, then it also converges at $x = 3$.