Math 350
Spring, 2006

**HOMEWORK #3**

Do 50 points of the following problems (due 2/3/06).

25 pts.  **1** Find a necessary condition on the length \( n \) so that the binary \((n, M, 3)\) code is perfect. What are the conditions for a perfect \( q \)-ary \((n, M, 3)\) code?

25 pts.  **2** Let \( a, b \in \mathbb{Z}_p \) for \( p \) a prime: show that \((a + b)^p \equiv a^p + b^p \mod p\). Explain how that can be extended to \((a + b + \cdots + z)^p \equiv a^p + b^p + \cdots + z^p \mod p\). Use this to show that \( x^p \equiv x \mod p \) for every \( x \in \mathbb{Z}_p \).

25 pts.  **3** Consider the following matrix:
\[
H = \begin{pmatrix}
0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 & 1
\end{pmatrix}
\]

Show that the set of vectors \( u = (u_1, u_2, \ldots, u_7) \) that satisfy \( Hu^T = (000) \) form a binary linear code. How many elements are there in this code? Use properties of the matrix \( H \) to determine the minimum distance of the code (don’t just use brute force).