Star problem for Thanksgiving

Show that the regular 9-gon cannot be constructed starting from $z_1 = 0$ and $z_2 = 1$ using only a straight-edge and a compass.

If we can show that $\eta = e^{2\pi i/9}$ has the property that $[\mathbb{Q}(\eta) : \mathbb{Q}] = 6$, then we will be able to conclude that the regular 9-gon cannot be constructed. We have that $\eta$ is a root of $x^9 - 1$, and we can factor this into $x^9 - 1 = (x - 1)(x^2 + x + 1)(x^6 + x^3 + 1)$. I claim that $x^6 + x^3 + 1$ is the minimal polynomial for $\eta$. We can easily check that $\eta^6 + \eta^3 + 1 = 0$, so it remains to show that $x^6 + x^3 + 1$ is irreducible. We can do this by using the mod2 test: over $GF(2)$, there are no linear factors since $0^6 + 0^3 + 1 = 1^6 + 1^3 + 1 = 1 \neq 0$. The only irreducible degree 2 polynomial is $x^2 + x + 1$, and it does not divide $x^6 + x^3 + 1$ (there is a remainder of 1 when you do the division). Finally, the only two irreducible degree 3 polynomials are $x^3 + x + 1$ and $x^3 + x^2 + 1$ and neither of them divides $x^6 + x^3 + 1$. Thus, this polynomial is irreducible mod2 and hence is irreducible over $\mathbb{Q}$ as required.