Sixth Homework Assignment

Please pledge this whole homework: no help from anyone (or any resources other than your book) for any of the problems.

Turn-in problems due 10/24: Chapter 18 8,18,20,26,32; Chapter 19 4,10,16,24,30

⋆ problem, 20 points: (Euclid’s algorithm for finding the greatest common divisor in a Euclidean domain). Let \(a_1, a_2\) be non-zero elements of a Euclidean Domain \(D\), where \(\delta\) is the measure function for \(D\). Define \(a_i\) and \(q_i\) recursively by 
\[
a_1 = q_1 a_2 + a_3, a_i = q_i a_{i+1} + a_{i+2}
\]
where \(\delta(a_{i+2}) = 0\) or \(\delta(a_{i+2}) < \delta(a_{i+1})\). Show that there exists an \(n\) such that \(a_n \neq 0\) but \(a_{n+1} = 0\), and that \(d = a_n\) is the greatest common divisor of \(a_1\) and \(a_2\). Also use the equations to obtain an expression for \(d\) in the form \(xa_1 + ya_2\) for \(x, y \in D\).