Fifth Homework Assignment

Turn-in problems due 10/6: Chapter 16 18,22,32,38,40,44; Chapter 17 12,16,28,30,32,34,36

⋆ problem, 20 pts.: Let $F[x_1, x_2, \ldots, x_r]$ be the ring of polynomials in $r$ variables over a field $F$.

a: If $F$ is finite and $r = 1$, show that there is a polynomial $f(x_1)$ so that $f(a) = 0$ for every $a \in F$. Show that if $r > 1$ and $F$ is finite that there is a polynomial $f(x_1, x_2, \ldots, x_r)$ so that $f(a_1, a_2, \ldots, a_r) = 0$ for every $(a_1, a_2, \ldots, a_r) \in F^r$.

b: If $F$ is infinite and $r = 1$, show that any nonzero polynomial $f(x_1)$ will have an $a \in F$ so that $f(a) \neq 0$. Use induction on the number of variables to argue that any nonzero polynomial $f(x_1, x_2, \ldots, x_r)$ must have elements $a_1, a_2, \ldots, a_r \in F$ satisfying $f(a_1, a_2, \ldots, a_r) \neq 0$. 