Quiz 4

1. A person with a headache takes 550mg of ibuprofen. Every four hours thereafter, she takes another 550mg of ibuprofen. During each four-hour period, her body removes 90% of the ibuprofen present in the blood at the start of the current four-hour period. Let $a_n$ denote the concentration of ibuprofen in her blood immediately before she takes the $n$th dose of ibuprofen.

   a. Write down a difference equation for $a_n$, and determine the value of $a_0$. (You may assume that this person's body contains 5.5 liters of blood.)

   $a_{n+1} - a_n = 100 \cdot 0.9 a_n$

   $a_{n+1} = 100 + 0.1 a_n$

   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   n & 0 & 1 & 2 & 3 & 4 & 5 \\
   \hline
   a_n & 100 & 110 & 111 & 111 & 111 & 111 \\
   \end{array}
   \]

   b. Compute the first five terms of the solution sequence.

   (4pts.) 2. Find $a_{12}$ if $a_{n+1} = \frac{a_n}{2}$, $a_0 = 1$.

   $a_{n+1} = \frac{a_n}{2}$

   \[
   \begin{array}{c|c|c|c|c|c|c|c}
   n & 0 & 1 & 2 & 3 & 4 & 5 \\
   \hline
   a_n & 1 & 0.5 & 0.25 & 0.125 & 0.0625 & 0.03125 \\
   \end{array}
   \]

   $a_{12} = \frac{2^{11} - 1}{2^{11}} = \frac{-2047}{2048}$

   (8pts.) 3. Find $\lim_{n \to \infty} a_n$ for the discrete dynamical system $a_{n+1} = \frac{1}{4}a_n + 2$, $a_0 = 3$. Showing all work.

   $a_{n+1} = Ma_n + b = M^n a_0 + b \sum_{i=0}^{n-1} M^i$

   In our case $M = \frac{1}{4}$ $b = 2$

   $\Rightarrow \lim_{n \to \infty} a_n = \lim_{n \to \infty} \left[ \left(\frac{1}{4}\right)^n (3) + 2 \sum_{i=0}^{n-1} \left(\frac{1}{4}\right)^i \right]$

   $= 2 \left( \frac{1}{1 - \frac{1}{4}} \right) = \frac{8}{3} = \left(2^{\frac{3}{2}}\right)$