For each function in Exercises 1-5, find the requested tangent polynomial at $x = 0$. In each case, use Mathematica to graph the function and the tangent polynomial on the same set of axes.

1. $f(x) = \sqrt{1 + x}$, tangent line.
2. $f(x) = \sin(x) + \cos(x)$, tangent parabola.
3. $f(x) = \frac{1}{1 - x}$, tangent parabola.
4. $f(x) = e^{-x^2}$, tangent parabola.
5. $f(x) = \sin(kx)$ ($k$ is a parameter), tangent parabola.

6. In class, we derived the following formulas for tangent polynomials of degrees one and two (i.e. tangent lines and tangent parabolas), to a function $f(x)$ at $x = 0$:

   $$p_1(x) = f(0) + f'(0)x$$
   $$p_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2$$

By starting with the generic cubic form $p_3(x) = a + bx + cx^2 + dx^3$, and mimicking what was done in class to derive the formula for $p_2$, derive a formula for the tangent polynomial of degree three to a function $f(x)$ at $x = 0$.

**What Can Tangent Polynomials Do for Me?**

In this and future assignments, we will be introduced to some of the uses for which tangent polynomials are particularly well-suited. Today’s application: Estimating values of functions at specific points, as illustrated by the following exercise.

7. (a) Compute the tangent polynomials of degrees one, two, and three, for the function $f(x) = e^x$ at $x = 0$.
   (b) Use the results of part (a) to compute three different estimates $e^{0.3}$. Comment on the accuracy of each.