Distribution Functions: Recall from class that a distribution function \( p(x) \) must satisfy the following properties:

- \( p(x) \geq 0 \) for each \( a \leq x \leq b \)
- \( \int_a^b p(x)dx = 1 \).

1. For each function graphed below, find a value of \( c \) so that the function is a distribution function.

2. After measuring the duration of many telephone calls, the telephone company found that their data was well-approximated by the distribution function \( p(x) = 0.4e^{-0.4x} \), where \( x \) is the duration of a call, in minutes.
   a. What fraction of calls last between 1 and 2 minutes?
   b. What fraction of calls last 1 minute or less?
   c. What fraction of calls last 3 minutes or more? (Exceedingly generous hint: (fraction of calls 3 or more minutes) = 1 – (fraction of calls less than 3 minutes).)

Cumulative Distribution Functions: If \( p(x) \) is a distribution (i.e. density) function on \([a,b]\), then the function \( P(x) = \int_a^x p(t)dt \) makes sense for any \( x \) in \([a,b]\). \( P(x) \) is called the cumulative distribution function for \( p(x) \) on \([a,b]\).

3. Let \( p(x) \) be the distribution function from Exercise 2.
   a. Compute \( P(2) \).
   b. In terms of phone calls, what does \( P(2) \) represent?

4. Use the properties of \( p(x) \) (listed at the top of this assignment), and the definition of \( P(x) \), to explain why the cumulative distribution function \( P(x) \) has each of the following properties:
   a. \( P(a)=0 \) and \( P(b)=1 \).
   b. \( P(x) \) is a non-decreasing function on \( a \leq x \leq b \).