Quiz 6

Davis
M231

Name:
Pledge:

(8pts.) 1. Use multivariate max/min techniques to find the line \( y = mx + b \) that minimizes the sums of squares of the errors to the points \{(0,0), (1,2), (2,3)\}.

\[ d^2 = f(m, b) = (b-o)^2 + (m+b-2)^2 + (2m+b-3)^2 \]

\[ f_m = 2(m+b-2)+2(2m+b-3)2 = 4m+6b-16 = 0 \]

\[ f_b = 2b+2(m+b-2)+2(2m+b-3) = 6m+6b-10 = 0 \]

\[ 4m-6=0 \Rightarrow m = \frac{3}{2} \Rightarrow b = \frac{1}{6} \]

\[ D = f_{mm}f_{bb} - f_{mb}^2 = 6(6) - \left( \frac{3}{2} \right)^2 > 0 \]

\[ f_{mm} > 0 \Rightarrow (m, b) = \left( \frac{3}{2}, \frac{1}{6} \right) \text{ is a min.} \]

(8pts.) 2. Use LaGrange multipliers to find the extreme values of \( f(x, y) = x^2 + 3y^2 \) subject to the constraint \( x^2 + y^2 = 4 \).

\[ \nabla f = \langle 2x, 6y \rangle \]

\[ \nabla g = \langle 2x, 2y \rangle \]

\[ \nabla f = \lambda \nabla g \Rightarrow 2x = \lambda 2x, \ 6y = \lambda 2y \]

\[ \Rightarrow \text{either } x = 0 \text{ or } \lambda = 1 \]

\[ \text{If } x = 0, \text{ then } x^2 + y^2 = 4 \text{ implies } y = \pm 2 \]

\[ \text{If } \lambda = 1, \text{ then } 6y = \lambda 2y \Rightarrow y = 0 \]

\[ \Rightarrow x = \pm 2 \]

\[ \text{Extrema values at } (\pm 2, 0) \text{ or } (0, \pm 2) \]

\[ (f_{\min}(\pm 2, 0) = 4, f(0, \pm 2) = 12 \max) \]

(6pts.) 3. A careless driver is going too fast, and she slams on her brakes when a deer runs across the road. Her speed after \( t \) seconds is given in the table below. Give a lower and an upper estimate of the distance travelled over the 6 seconds.

<table>
<thead>
<tr>
<th>( t )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v )</td>
<td>60</td>
<td>30</td>
<td>15</td>
<td>5</td>
</tr>
</tbody>
</table>

\[ LHS = 60(2) + 30(4) + 15(6) = 210 \text{ ft} \]

\[ RHS = 30(2) + 15(4) + 5(6) = 100 \text{ ft} \]

Upper estimate
Lower estimate