1. a. \[ \int \sin^2(e^x) \cos(e^x) e^x \, dx = \int \sin^2 u \cos u \, du = \int v^2 \, dv \]
\[
\begin{align*}
  u &= e^x \\
  du &= e^x \, dx \\
  v &= \sin u \\
  dv &= \cos u \, du \\
  &= \frac{v^3}{3} + C \\
  &= \frac{\sin^3 u}{3} + C \\
  &= \frac{\sin^3 (e^x)}{3} + C
\end{align*}
\]

b. \[ \int e^x \sin x \, dx = -e^x \cos x + \int e^x \cos x = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx \]
\[
\begin{align*}
  u &= e^x \\
  v &= \sin x \\
  u' &= e^x \\
  v' &= \cos x \\
  2 \int e^x \sin x \, dx &= e^x (-\cos x + \sin x) \\
  \int e^x \sin x \, dx &= \frac{e^x}{2} (-\cos x + \sin x) + C
\end{align*}
\]

C. \[ \int \frac{2x-7}{x^2-7x+6} \, dx = \int \frac{du}{u} = \ln |u| + C = \ln |x^2-7x+6| + C \]
\[
\begin{align*}
  u &= x^2-7x+6 \\
  du &= (2x-7) \, dx \\
  \frac{2x-7}{x^2-7x+6} &= \frac{A}{x-6} + \frac{B}{x-1} \\
  2x-7 &= A(x-1) + B(x-6) \\
  x=1: & \quad -5 = B(-5) \quad B=1 \\
  x=6: & \quad 5 = A(5) \quad A=1 \\
\end{align*}
\]

\[ \int \frac{2x-7}{x^2-7x+6} \, dx = \int \left( \frac{1}{x-6} + \frac{1}{x-1} \right) \, dx = \ln |x-6| + \ln |x-1| + C \]
2. a. \(-\sin x + n \int x \cos x \ dx + C\)

3. a. \(\int_{x_1}^{x_2} e^{-x^2} \ dx\)

b. \(\left[\frac{e^{-x^2}}{-2}\right]_{x_1}^{x_2} = \frac{e^{-x_2^2} - e^{-x_1^2}}{2}\)

\(\text{The area } S \text{ of } 0.15 \text{ of } 1.49\).

The trapezoids lie above the curve, so \(T \geq S\).

The trapezoids are below the line for \(0 \leq x \leq 0.738\).
4. a. \[ S_0^2 \frac{1}{(x-1)^2} \, dx = S_0^1 \frac{1}{(x-1)^2} \, dx + S_1^2 \frac{1}{(x-1)^2} \, dx \]

Do separately: \[ S_0^1 \frac{1}{(x-1)^2} \, dx = \lim_{t \to 1^-} \left[ S_t^0 \frac{1}{(x-1)^2} \, dx \right] \]

\[ = \lim_{t \to 1^-} \left[ -\frac{1}{(x-1)} \right]_0^t = \lim_{t \to 1^-} \left[ -\frac{1}{(t-1)} + 1 \right] = \infty \]

DIVERGES

This limit is infinite since \( \frac{1}{t-1} \) has a zero in the denominator when we apply the limit (so the fraction blows up).

b. \[ S_2^\infty \frac{1}{\sqrt{x-1}} \, dx = \lim_{t \to \infty} \left[ S_t^2 \frac{1}{\sqrt{x-1}} \, dx \right] > \lim_{t \to \infty} \left[ S_2^t \frac{1}{\sqrt{y-1}} \, dy \right] \]

\[ = \lim_{t \to \infty} \left[ 2 \sqrt{x} \right]_{2}^{t} = \lim_{t \to \infty} \left[ 2 \sqrt{t} - 2 \sqrt{2} \right] \]

DIVERGES by the comparison test, since a smaller function has an infinite area under it.

5. Intersection points:

\[ y^2 + y + 1 = -y^2 - 3y + 31 \]
\[ 2y^2 + 4y - 30 = 0 \]
\[ y^2 + 2y - 15 = 0 \]
\[ y + 5 \cdot (y - 3) = 0 \]
\[ y = -5, 3 \]

Right-left:

\[ S_{-5}^3 \left( -y^2 - 3y + 31 - (y^2 + y + 1) \right) \, dy \]

\[ = S_{-5}^3 \left( -2y^2 - 4y + 30 \right) \, dy \]

\[ = -\frac{2}{3} y^3 - 2y^2 + 30y \bigg|_{-5}^{3} \]

\[ = -18 - 18 + 90 - \left( \frac{250}{3} - 50 - 15 \right) \]

\[ = 170 \frac{2}{3} \]