Quiz 6

Show all work: unjustified answers may receive less than full credit.

1. Find the derivative of the following functions:
   (a) \( f(x) = 3x^2 - 2\cos(x) \)
   \[ f'(x) = 6x + 2\sin(x) \]
   (b) \( F(x) = (x^4 + 3x^2 - 2)^5 \)
   \[ F'(x) = 5(x^4 + 3x^2 - 2)^4 (4x^3 + 6x) \]
   (c) \( y = \sin(a^3 + x^3) \)
   \[ y' = \cos(a^3 + x^3)(3x^2) \]
   (d) \( x^2 + xy - y^2 = 4 \)
   \[
   2x + xy' + y - 2yy' = 0 \\
   y'(x-2y) = -y - 2x \\
   y' = \frac{y - 2x}{x - 2y}
   \]

2. Find an equation of the tangent line to the curve \( y = x + \cos(x) \) at the point \((0, 1)\).
   \[
   y' = 1 - \sin(x) \\
   y'(0) = 1 \\
   y - 1 = 1(x - 0)
   \]

3. Use the definition of the derivative to show that the derivative of \( y = x^n \) is \( y' = nx^{n-1} \). Show where you used the assumption that \( n \) is a positive integer.
   \[
   (x^n)' = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \text{junk}(h^2) - x^n}{h} \\
   = \lim_{h \to 0} \frac{nx^{n-1}h + \text{junk}(h^2)}{h} = nx^{n-1}
   \]

   \text{This equality uses binomial expansion (Pascal's triangle).}