Quiz 10

Show all work: unjustified answers may receive less than full credit.

1. Estimate the area under the graph of \( f(x) = \cos(x) \) from \( x = 0 \) to \( x = \pi/2 \) using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

\[
A \approx \frac{\pi}{8} \left( \cos \left( \frac{\pi}{8} \right) + \cos \left( \frac{3\pi}{8} \right) + \cos \left( \frac{5\pi}{8} \right) + \cos \left( \frac{7\pi}{8} \right) \right)
\]

\[
= 0.79076626
\]

This estimate is an underestimate.

2. Oil leaked from a tank at a rate of \( r(t) \) liters per hour. The rate decreased as time passed and values of the rate at two-hour time intervals are shown in the table. Find lower and upper estimates for the total amount of oil that leaked out.

<table>
<thead>
<tr>
<th>t (h)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t) ) (L/h)</td>
<td>8.7</td>
<td>7.6</td>
<td>6.8</td>
<td>6.2</td>
<td>5.7</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Upper estimate: \( 2 \left( 8.7 + 7.6 + 6.8 + 6.2 + 5.7 \right) \)

\[
= 70 \text{ liters}
\]

Lower estimate: \( 2 \left( 7.6 + 6.8 + 6.2 + 5.7 + 5.3 \right) \)

\[
= 63.02 \text{ liters}
\]