TEST 3

Davis 
Name: 
M211 
Pledge: 

Show all work: unjustified answers may receive less than full credit.

(20pts.) 1. A piece of wire 10 m long is cut into two pieces. Both pieces are bent into squares. How should the wire be cut so that the total area enclosed is a minimum? (Show all steps!)

(20pts.) 2. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is $800 per month. A market survey indicates that, on average, one additional unit will remain vacant for each $10 increase in rent. The cost of operating the complex is $C(x) = 60000 + 100x$, where $x$ is the number of units occupied.
   a. Find the demand function, assuming it is linear.
   b. What rent should the manager charge to maximize profit?

(20pts.) 3. Sketch a plot of $f(x) = x^4 - 4x^3$ showing all work (you may NOT use your calculator graphing features for this problem).

(15pts.) 4. Find $f(x)$ if $f''(x) = \sin(x) + \cos(x), f(0) = 3, f'(0) = 4$

(15pts.) 5. Use two iterations of Newton’s method (calculate $x_3$) to estimate $\sqrt{31.7}$.

(10pts.) 6. Estimate the area under the graph of $f(x) = \frac{1}{1+x^2}$ from $x = -1$ to $x = 1$ using four rectangles and right endpoints.
1. \[ \begin{array}{c}
\frac{x}{2} \\
\frac{y}{4}
\end{array} \]

Step 2: Minimize area of two squares

Step 3: \[ A = \left( \frac{x}{4} \right)^2 + \left( \frac{y}{4} \right)^2 = \frac{1}{16} (x^2 + y^2) \]

Step 4: \[ x + y = 10 \Rightarrow y = 10 - x \]
\[ A = \frac{1}{16} (x^2 + (10-x)^2) \]

Step 5: \[ A' = \frac{1}{16} \left[ 2x - 2(10-x) \right] = 0 \]
\[ \frac{1}{16} \left[ 4x - 20 \right] = 0 \]
\[ 4x = 20 \]
\[ x = 5 \]

Cut the wire in half to minimize area.

2. (a) \[ m = \frac{810-800}{99-100} = -10 \]
\[ b = -800 = -10(x-100) \]
\[ \rho = -10x + 1800 \]

(6) Maximize profit: \[ \Pi = R - C = -10x^2 + 1800x - 600000 \]
\[ \Pi' = -20x + 1700 = 0 \]
\[ x = \frac{1700}{20} = 85 \]
\[ \rho = -10(85) + 1800 = 950 \]

Change $950 monthly rent to maximize profit.
3. \( y = x^4 - 4x^3 \)
   \[= x^3(x - 4)\]
   \( x = 0, 4 \) are x-intercepts

\( y' = 4x^3 - 12x^2 \)
\[= 4x^2(x - 3)\]
\( x = 0, 3 \) are critical pts

\( y'' = 12x^2 - 24x \)
\[= 12x(x - 2)\]

4. \( f''(x) = \sin(x) + \cos(x) \)
   \( f'(x) = -\cos(x) + \sin(x) + C \)

4 = \( f'(0) = -\cos(0) + \sin(0) + C \)
\[= 5 \]

\( f'(x) = -\cos(x) + \sin(x) + 5 \)

\( f(x) = -\sin(x) - \cos(x) + 5x + C' \)

3 = \( f(0) = -\sin(0) - \cos(0) + 5(0) + C' \)
\[= 4 \]

\( f(x) = -\sin(x) - \cos(x) + 5x + 4 \)
5. \( x = \sqrt[5]{31.7} \Rightarrow x^5 = 31.7 \Rightarrow f(x) = x^5 - 31.7 \)

Initial guess: \( x_1 = 2 \) (since \( \sqrt[5]{32} = 2 \))

\[
x_2 = 2 - \frac{2^5 - 31.7}{5(2)^4} = 2 - \frac{32 - 31.7}{80} =
\]

6. 

\[
R_4 = \frac{1}{2} \left[ \frac{1}{1 + (\frac{1}{2})^2} + \frac{1}{1 + 0^2} + \frac{1}{1 + (\frac{1}{2})^2} + \frac{1}{1 + (1)^2} \right]
\]