Quiz 6

Show all work; unjustified answers may receive less than full credit.

(8pts.) 1. A store selling gasoline at $3.25 per gallon sells 1000 gallons per day. On Thrifty Thursdays they reduce the price by $1.10 per gallon and sell 1100 gallons that day.

a. Find the demand function assuming it is linear.

\[ p(x) = 3.25 - 0.001(x - 1000) \]
\[ \bar{p}(x) = 3.25 - 0.001x + 1.25 \]

b. If the shop's weekly cost function is \( c(x) = 5000 + 2x \), how much should they reduce their price in order to maximize profit?

\[ \pi(x) = -0.002x + 2.25 = 0 \]
\[ x = \frac{2.25}{0.002} = 1125 \text{ gallons} \]
\[ \pi(1125) = -1.25 + 4.25 = 3.025 \text{ dollars per gallon} \]

(6pts.) 2. Calculate the following limits.

a. \( \lim_{x \to 0} \frac{e^x - 1}{x} = \lim_{x \to 0} \frac{e^x}{1} = \lim_{x \to 0} \frac{1}{1} = 1 = \frac{1}{2} \)

b. \( \lim_{x \to 0^+} \sqrt{x} \ln(x) \)

\[ \lim_{x \to 0^+} \frac{\ln(x)}{\sqrt{x}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{1}{2}\sqrt{x}} = \lim_{x \to 0^+} 2 \cdot \frac{1}{\sqrt{x}} = \lim_{x \to 0^+} 2 \cdot \frac{1}{2} = 0 \]

(6pts.) 3. Use two iterations of Newton's method (calculate \( x_3 \)) to estimate \( \sqrt{13} \).

\[ f(x) = x^2 - 13 \]
\[ f'(x) = 2x \]
\[ x_1 = 3.5 \]
\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 3.5 - \frac{(3.5)^2 - 13}{2(3.5)} = 3.5 - \frac{75}{7} = 3.5625 \]
\[ x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 3.5625 - \frac{(3.5625)^2 - 13}{2(3.5625)} = 3.6055 \]

\( \sqrt{13} \approx 3.6055 \)