Quiz 3

Show all work: unjustified answers may receive less than full credit.

(8pts.) 1. Use the definition of the derivative to show that \((x^n)' = nx^{n-1}\) for \(n\) a positive integer. Explain where your argument uses the fact that \(n\) is a positive integer.

\[
(x^n)' = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \text{junk}(h^2) - x^n}{h} = \lim_{h \to 0} \frac{nx^{n-1}h}{h} = nx^{n-1}
\]

(to expand \((x+h)^n\) we need \(n\) to be a positive integer (Foil only works for positive integers)

(6pts.) 2. Calculate \(\frac{du}{dx}\) for the following functions.

a) \(y = 3x^9 - \sqrt{5}/x^3\)
\[
\frac{dy}{dx} = 27x^8 + \frac{3}{5\sqrt{5}}x^{-\frac{8}{5}}
\]

b) \(y = \frac{2x^2 + 5x + 7}{\sqrt{x}}\)
\[
\frac{dy}{dx} = 2x + \frac{5}{2}\sqrt{x} \cdot x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = 3x^\frac{1}{2} + \frac{5}{2}x^{-\frac{1}{2}} = \frac{7}{2}x^{-\frac{3}{2}}
\]

c) \(y = \frac{(x^2 + 3x)e^x}{x^3 - 6x^3}\)
\[
\frac{dy}{dx} = \frac{(x^5 - 6x^3)(x^2 + 3x)e^x + e^x(2x + 3) - (x^2 + 3x)e^x [5x^4 - 18x^3]}{(x^5 - 6x^3)^2}
\]

(6pts.) 3. The average cost of manufacturing a quantity \(q\) of a good is \(a(q) = \frac{C(q)}{q}\), where \(C(q) = .03q^2 - .4q + 23\) is the cost (in thousands of dollars) of producing \(q\) (in hundreds of items). Calculate \(a(20)\) and \(a'(20)\) and explain to your manager what those numbers represents.

\[
a(q) = \frac{C(q)}{q} = \frac{.03q^2 - .4q + 23}{q} = .03q - .4 + 23q^{-1}\]
\[
a(20) = .03\cdot 20 - .4 + 23\cdot 20^{-1} = .6\]

\[
a'(q) = .03 - 23q^{-2} - a'(20) = .03 - \frac{23}{20^2} = -.0275
\]

When 2000 items are produced, the average cost is $1.350 per hundred items.

When producing 2000 items, the average cost is decreasing by $27.50 per hundred items.