Show all work; unjustified answers may receive less than full credit.

(20pts.) 1. A right circular cylinder is inscribed in a cone with height \( h \) and base radius \( r \). Find the largest possible volume of such a cylinder.

(20pts.) 2. The manager of a 100-unit apartment complex knows from experience that all units will be occupied if the rent is $800 per month. A market survey indicates that, on average, one additional unit will remain vacant for each $10 increase in rent. The cost of operating the complex is \( C(x) = 60000 + 100x \), where \( x \) is the number of units occupied.

   a. Find the demand function, assuming it is linear.

   b. What rent should the manager charge to maximize profit?

(15pts.) 3. Sketch a plot of \( f(x) = x^4 - 4x^3 \) showing all work (you may NOT use your calculator graphing features for this problem).

(15pts.) 4. Find \( f(x) \) if \( f''(x) = x^{-2}, x > 0, f(1) = 0, f'(1) = 3 \)

(15pts.) 5. Use two iterations of Newton's method (calculate \( x_3 \)) to estimate \( \sqrt{31.7} \).

(15pts.) 6. a. Estimate the area under the graph of \( f(x) = \frac{1}{1+x^2} \) from \( x = -1 \) to \( x = 1 \) using four rectangles and right endpoints.

   b. If \( v(t) = \frac{1}{1+t^2} \) is the velocity of a particle, find the distance traveled from \( t = -1 \) to \( t = 1 \).
Maximize Volume of cylinder
\[ V = \pi R^2 H \]
\[ V = \frac{\pi}{H} \left[ (h-H)^2 H \right] \]
\[ V' = \frac{\pi}{H} \left[ (h-H)^2 + (-2)(h-H)H \right] = 0 \]
\[ (h-H)[h-H-2H] = 0 \]
\[ H = h, \quad H = \frac{2h}{3} \]

\[ V = \frac{\pi}{h} \left[ (h-H)^2 \right]^{\frac{1}{3}} \]
\[ = \frac{4}{27} \pi r^2 h \]

2. \[ M = \frac{810 - 800}{99 - 100} = -10 \]
\[ p - 800 = -10(x - 100) \]
\[ p = -10x + 1800 \]

(b) \[ R(x) = xp = -10x^2 + 1800x \]
\[ \Pi(x) = -10x^2 + 1800x - (60000 + 100x) \]
\[ = -10x^2 + 1700x - 60000 \]
\[ \Pi'(x) = -20x + 1700 = 0 \]
\[ x = 85 \Rightarrow p = -10(85) + 1800 = 950 \]

Charge $950 to maximize profit.
3. \( y = x^4 - 4x^3 = (x - 4)x^3 \)
\[ y' = 4x^3 - 12x^2 = 4x^2(x - 3) \]
\[ x = 0 \text{ crit pts.} \]
\[ y'' = 12x^2 - 24x = 12x(x - 2) \]
\[ x = 0, 1 \text{ possible infl pts.} \]

4. \( f''(x) = x^{-2} \)
\[ f'(x) = -x^{-1} + C \]
\[ = -x^{-1} + 4 \]
\[ f(x) = -\ln|x| + 4x + C \]
\[ O = -\ln|1| + 4(1) + C \]
\[ C = -4 \]
\[ f(x) = -\ln|x| + 4x - 4 \]

5. \( f(x) = x^5 - 31.7 \)
\[ f'(x) = 5x^4 \]
\[ x_1 = 2 \]
\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2 - \frac{3}{80} = 1.99625 \]
\[ x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.99624 \]
\[ \sqrt[5]{31.7} \approx 1.9962358 \]
6. a. \[ \frac{1}{4} \left[ f(-.5) + f(0) + f(.5) + f(1) \right] \]

\[ = \frac{1}{2} \left[ \frac{1}{1 + (-.5)^2} + \frac{1}{1 + 0^2} + \frac{1}{1 + (.5)^2} + \frac{1}{1 + 1^2} \right] \]

\[ = \frac{1}{2} \left[ \frac{31}{10} \right] = \frac{31}{20} = 1.55 \]

b. \[ s(t) = \tan^{-1}(t) + C \]

\[ s(1) = \tan^{-1}(1) + C = \frac{\pi}{4} + C \]

\[ s(-1) = \tan^{-1}(-1) + C = -\frac{\pi}{4} + C \]

distance traveled = change in position

\[ = \frac{\pi}{4} + C - (-\frac{\pi}{4} + C) = \frac{\pi}{2} \]