Quiz 3

Show all work: unjustified answers may receive less than full credit.

(6pts.)
1. Use the definition of the derivative to show that \((x^n)' = nx^{n-1}\) for \(n\) a positive integer.

\[
(x^n)' = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h \to 0} x^n + nx^{n-1}h + \text{junk}(h) - x^n
\]

\[
= \lim_{h \to 0} \frac{nx^{n-1}h + \text{junk}(h)}{h} = nx^{n-1}
\]

(8pts.)
2. Calculate \(\frac{du}{dx}\) for the following functions.

a. \(y = 5x^6 - \sqrt{2}/x^3\) \(y' = 30x^5 + \frac{2\sqrt{2}}{3}x^{-\frac{5}{2}}\)

b. \(y = e^x \sin(x)\)  
\(y' = e^x \cos(x) + e^x \sin(x)\)

c. \(y = \frac{x^3 + 4x + 3}{\sqrt{x}}\)  
\(y' = \frac{3x^2 + 4}{2x^{\frac{1}{2}}} + 3x^{-\frac{1}{2}} \Rightarrow y' = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} = 0 - \frac{3}{2}x^{-\frac{3}{2}}\)

d. \(y = \frac{xe^x}{x^2 - 6}\)  
\(y' = \frac{(x^2 - 6)(xe^x - e^x) - xe^x(2x)}{(x^2 - 6)^2}\)

(6pts.)
3. The average cost of manufacturing a quantity \(q\) of a good is \(a(q) = \frac{C(q)}{q}\), where \(C(q) = .01q^2 - .6q + 13\) is the cost (in millions of dollars) of producing \(q\). Calculate \(a'(20)\) and explain to your manager what that number represents.

\[
a(q) = \frac{C(q)}{q} = \frac{.01q^2 - .6q + 13}{q}
\]

\[
a'(q) = .01 - \frac{.13}{q^2} = .01 - \frac{13}{400} = -.0225
\]

When we are manufacturing \(q = 20\) objects, the average cost of producing more will be \(-.0225\) (in millions of dollars) less per extra unit.