Show all work: unjustified answers may receive less than full credit.

(25pts.) 1. A fence 5 feet tall runs parallel to a tall building at a distance of 5 feet from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

(25pts.) 2. A pizza shop has been selling 500 pizzas a week at $10 a pizza. A customer survey indicates that for each $2.25 in coupons offered to the buyer, the number of pizzas sold will increase by 50 pizzas per week.
   a. Find the demand function.
   b. If the shop’s weekly cost function is \( c(x) = 1000 + 5x \), how big should they make their coupon in order to maximize profit?

(20pts.) 3. Sketch a plot of \( f(x) = x^{1/3}(x - 2) \) showing all work (you may NOT use your calculator graphing features for this problem).

(10pts.) 4. Calculate the following limits.
   a. \( \lim_{x \to 0^+} \frac{\tan x}{x^2} \)
   b. \( \lim_{x \to 0^-} \sqrt{x} \ln(x) \)

(20pts.) 5. Use two iterations of Newton’s method (calculate \( x_3 \)) to estimate the positive root of \( \cos(x) = x \). (Hint: draw a picture to show how you would estimate \( x_1 \). I will give you a value for \( x_1 \) if you don’t know how to do that for 5 points.).
1. \[ \begin{align*}
\text{Minimize } l \text{ (Actually minimize } l^2) \\
\text{(Pythagorean Thm)}
\end{align*} \]

\[ l^2 = x^2 + y^2 \]

\[ \frac{y}{5} = \frac{x}{x-5} \Rightarrow y = \frac{5x}{x-5} \]

\[ l^2 = x^2 + \frac{25x^2}{(x-5)^2} \]

\[ \frac{d(l^2)}{dx} = 2x + \frac{2x(50x-50x^2)}{(x-5)^3} = 0 \]

\[ 2x(x-5) = \frac{25}{x-5} \]

\[ x-5 = 5 \Rightarrow x = 10 \]

\[ l^2 = 10^2 + 10^2 = 200 \Rightarrow l = 10\sqrt{2} \]

2. \[ \begin{align*}
y - 10 &= M(x-500) \\
m &= \frac{10-9}{500-700} = \frac{1}{200} \\
(M) y &= 10 - \frac{1}{200} (x-500) = -\frac{1}{200} x + 12.55 \\
(p(x)) &= x p(x) = y - C(x) = -\frac{1}{200} x^2 + 7.50 x - (1000 + 5x) \\
&= -\frac{1}{200} x^2 + 7.50 x - 1000 \\
p'(x) &= -\frac{1}{200} x + 7.50 = 0 \Rightarrow x = 750 \\
\text{Price: } p(750) &= -\frac{1}{200}(750) + 12.50 = \$8.75 \\
\text{Coupon: } 10 - $8.75 = $1.25
\end{align*} \]
3. \( f(x) = x^{\frac{5}{3}}(x-2) \)  
   \( x \)-intercepts \( x=0, 2 \)  
   \( f'(x) = \frac{4}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3}x^{-\frac{1}{3}}(2x-1) \)  
   crit pts \( x=0, \frac{1}{2} \)  
   \( f''(x) = \frac{4}{9}x^{-\frac{5}{3}} + \frac{4}{9}x^{-\frac{5}{3}} = \frac{8}{9}x^{-\frac{5}{3}}(x+1) \)  
   poss. infl pts \( x=0, -1 \)  

4. (a) \( \lim_{t \to 0^+} \frac{e^t-1}{t^2} = \lim_{t \to 0^+} \frac{e^t}{2t} = +\infty \)

   (b) \( \lim_{x \to 0^+} \frac{\ln(x)}{x^{-\frac{1}{2}}} = \lim_{x \to 0^+} \frac{\frac{1}{x} - \frac{1}{2}x^{-\frac{3}{2}}}{\frac{1}{2}x^{-\frac{3}{2}}} = \lim_{x \to 0^+} \frac{\frac{x^2}{2}}{\frac{1}{2}} = 0 \)

5. \( f(x) = \cos x - x \)  
   \( f'(x) = -\sin x - 1 \)  
   \( x_1 = 1 \)  
   \( x_2 = 1 - \frac{\cos(1)-1}{-\sin(1)-1} = .7504 \)  
   \( x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = .7391 \)