I: Multiple Choice

1: Which of the following statements about a connected graph is always true?
   a: Every pair of vertices is joined by a single edge.
   b: A path of edges exists between any two vertices of the graph.
   c: There are an even number of vertices on the graph.
   d: There are an even number of edges on the graph.

   B

2: For the traveling salesman problem applied to five cities, how many distinct tours are possible?
   a: 120
   b: 60
   c: 24
   d: 12

   D

3: What is the minimum time required to complete twelve independent tasks on three processors when the sum of all the times of the nine tasks is 108 minutes?
   a: 36 minutes
   b: 28 minutes
   c: 124 minutes
   d: 12 minutes

   A

4: Write a resource constraint for this situation: producing a plastic ruler ($x$) requires 10 grams of plastic while producing a pencil box ($y$) requires 30 grams of plastic. There are 2000 grams of plastic available.
   a: $200x = (2000/30)y \leq 2000$
   b: $30x + 10y \leq 2000$
   c: $10x + 30y \leq 2000$
   d: $x + y \leq 2000$

   C

5: The corners of a graph of the feasible region for a mixture problem are $\{(0, 0), (0.5), (3.4), (7, 2), (9, 0)\}$. Find the point that maximizes the profit function $P = 2x + 5y$.

   a: (0.5)
   b: (3.4)
   c: (7.2)
d: (9, 0)

6: Chris and Terry must make a fair division of a stereo, a television, and a microwave. They have assigned points to the objects as shown below. Using the adjusted winner procedure, what does Terry receive?

<table>
<thead>
<tr>
<th>Object</th>
<th>Chris's points</th>
<th>Terry's points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stereo</td>
<td>25</td>
<td>45</td>
</tr>
<tr>
<td>Television</td>
<td>60</td>
<td>35</td>
</tr>
<tr>
<td>Microwave</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

a: Stereo and 1/4 of microwave
b: Stereo and 6/7 of microwave
c: Microwave and 6/7 of television
d: Stereo and microwave

7: Jack and Jill went up the hill and found an antique water pail. Jack and Jill must now make a fair division of the pail using the Knaster inheritance procedure. Jack bids $80 for the value of the pail and Jill bids $65. How much does Jill think she ends up with?

a: $36.25  
b: $32.50  
c: $40  
d: $43.75

8: A county is divided into three districts with the populations as shown below. They are to apportion 10 seats on the county council. Find the apportionment for Applewood.

<table>
<thead>
<tr>
<th>District</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applewood</td>
<td>8280</td>
</tr>
<tr>
<td>Boxwood</td>
<td>4600</td>
</tr>
<tr>
<td>Central</td>
<td>5220</td>
</tr>
</tbody>
</table>

A: $2800 = 4.57  
B: $4600 = 2.54  
C: $5220 = 2.88

9: We are scheduling seven course sections for a total of 217 students. Enrollments are: 109 in calculus 1, 79 in calculus 2, and 29 in advanced calculus. Find the apportionment for each course using the Jefferson method.

a: 4.2.1  
b: 3.3.1  
c: 4.3.0  
d: 3.2.2
10: In the following two-person game, the payoffs represent gains to the row Player 1 and losses to the column Player 2. Which of the following is true?

\[
\begin{bmatrix}
1 & 3 \\
4 & 2
\end{bmatrix}
\]

a: The game has no saddle point  
b: The value 3 is a saddle point  
c: The value 2 is a saddle point  
d: The value 2.5 is a saddle point

20pts. apiece

II: Free response:
1: Identify an Euler circuit on the graph below by numbering the sequence of edges in the order traveled.

2: Find an eulerization with 9 added edges for a 3-by-6 block rectangular street network.

3: In some states, license plates use a mixture of letters and numerals. How many possible plates could be constructed using three letters followed by three numerals?

\[26^3 \cdot 10^3 = 17,576,000\]
4: Given the order-requirement digraph below (with time given in minutes), apply the critical path scheduling algorithm to construct a schedule using three processors.

5: A small stereo manufacturer makes a receiver and a CD player. Each receiver takes 8 hours to assemble, 1 hour to test and ship, and earns a profit of $30. Each CD player takes 15 hours to assemble, 3 hours to test and ship, and earns a profit of $50. There are 160 hours available in the assembly department and 27 hours available in the testing and shipping department. What should the production schedule be to maximize profit?

6: Apply the Northwest Corner Rule to the following tableau. Make two modifications to the distribution that leave the rim numbers fixed but decrease the cost (explain in one sentence what you did).
7: In recent labor-management negotiations, several issues were identified, and points assigned to them indicating relative importance to each side, as below. Use the adjusted winner procedure to determine a fair resolution between labor and management.

<table>
<thead>
<tr>
<th>Issue</th>
<th>Management’s points</th>
<th>Labor’s points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base pay</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>Incentive pay</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Health care benefits</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Worker safety</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Opportunity for promotion</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>Retirement package</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Employee accountability</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

\[ x = \% \text{ of retirement management gets} \]

\[
46 + 20x = 50 + 20(1-x) \\
46 + 20x = 50 + 20 - 20x \\
40x = 24 \\
x = \frac{3}{5} \\
\]

Management gets incentive pay, promotion, accountability, & 60% of retirement. Labor gets rest. Both have 58 pts.

8: Four people must make a fair division of two classic cars. They use the Knaster inheritance procedure. Their bids are shown below. What is the division of the objects?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Bird</td>
<td>60,000</td>
<td>80,000</td>
<td>60,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Mustang</td>
<td>80,000</td>
<td>60,000</td>
<td>80,000</td>
<td>100,000</td>
</tr>
</tbody>
</table>

kitty = 60,000 + 75,000 = 135,000

\[
\frac{20}{35} + \frac{15}{15} + \frac{20}{35} + \frac{10}{10} = \frac{35}{35} = 1 \\
\]

Draw on kitty:

\[
\frac{A}{15} \frac{B}{15} \frac{C}{10} \frac{D}{10} \]

\[
\frac{20}{35} + \frac{15}{15} + \frac{20}{35} + \frac{10}{10} = 95,000 \\
\]

A gets $45,000; B gets T-Bird; owes $35,000; C gets $45,000; D gets Mustang, owes $55,000
9: Given the cities and the populations below, use the Hamilton method of apportionment to distribute the 25 seats on a regional board.

<table>
<thead>
<tr>
<th>City</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greenville</td>
<td>34,569</td>
</tr>
<tr>
<td>Riverdale</td>
<td>27,943</td>
</tr>
<tr>
<td>Oceanside</td>
<td>21,350</td>
</tr>
<tr>
<td>Parkview</td>
<td>16,138</td>
</tr>
</tbody>
</table>

\[
\text{District size } \frac{100,000}{25} = 4,000
\]

\[
G: \frac{34,569}{4,000} = 8.64225
\]

\[
R: \frac{27,943}{4,000} = 6.9\ldots
\]

\[
O: \frac{21,350}{4,000} = 5.3\ldots
\]

\[
P: \frac{16,138}{4,000} = 4.03\ldots
\]

Initial: 8+6+5+4 = 23 (2 extra)

Riverdale gets 1st, Greenville gets 2nd.

Greenville 9
Riverdale 7
Oceanside 5
Parkview 4

10: Solve the following game of batter vs. pitcher in baseball.

Show both the strategy the batter should follow as well as the strategy the pitcher should follow.

|          | Pitcher
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fastball</td>
</tr>
<tr>
<td>Batter</td>
<td>0.300</td>
</tr>
<tr>
<td>Knuckleball</td>
<td>0.100</td>
</tr>
</tbody>
</table>

\[
\rho = \% \text{ Fastball}
\]

Batter expects fastball

\[
E_F = 0.3\rho + 0.2(1-\rho)
\]

Batter expects knuckleball

\[
E_K = 0.1\rho + 0.3(1-\rho)
\]

\[
E_F = E_K
\]

\[
0.3\rho + 0.2(1-\rho) = 0.1\rho + 0.3(1-\rho)
\]

\[
0.3\rho + 0.2 = 0.1\rho + 0.3
\]

\[
0.2 = 0.2\rho
\]

\[
\rho = 1\frac{1}{5}
\]

\[
q = \% \text{ expect fastballs}
\]

0 = \% expect fastballs

\[
P\text{itcher throws fastball } E_F = 0.3q + 1(1-q)
\]

Pitcher throws knuckleball

\[
E_K = 0.2q + 0.3(1-q)
\]

\[
0.3q + 1-q = 0.2q + 3-0.3q
\]

\[
0.2 = 0.2q
\]

\[
q = \frac{1}{5}
\]

Batter should expect fastball \(\frac{2}{5}\) of time, knuckleball \(\frac{3}{5}\) of time.

\[
P\text{itcher should throw } \frac{1}{5} \text{ Fastballs, } \frac{3}{5} \text{ knuckleballs}
\]