1: Multiple Choice
   1: Which of the following defines the valence of vertex A of a graph?
      a: The total number of vertices of the graph
      b: The number of edges meeting at vertex A
      c: The total number of edges in the graph
      d: None of the above

2: For the traveling salesman problem applied to seven cities, how many distinct tours are possible?
   a: 360
   b: 720
   c: 2520
   d: 5040

3: What is the minimum time required to complete nine independent tasks on three processors when the sum of all the times of the nine tasks is 72 minutes?
   a: 3 minutes
   b: 8 minutes
   c: 24 minutes
   d: 27 minutes

4: Write a resource constraint for this situation: A lawn service company has 40 hours of worker time available. Mowing a lawn (x) takes 3 hours and trimming (y) takes 2 hours. The profit from mowing is $15 and the profit from trimming is $10.
   a: $3x + 2y \leq 40$
   b: $\frac{3}{2}x + 10y \leq 40$
   c: $15x + 10y \leq 40$
   d: $5x + 5y \leq 40$

5: The corners of a graph of the feasible region for a mixture problem are $\{(0,0), (0,8), (6,6), (10,2), (12,0)\}$. Find the point that maximizes the profit function $P = 3x + 6y$.
   a: (0,8)
   b: (6,6)
   c: (10,2)
   d: (12,0)
6: Tom and Sandy must make a fair division of three objects left by their Great Aunt Sally. They have assigned points to the objects as shown below. Using the adjusted winner procedure, how many points of value does Tom feel he ends with?

<table>
<thead>
<tr>
<th>Object</th>
<th>Tom's points</th>
<th>Sandy's points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Painting</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>Jewelry</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Car</td>
<td>50</td>
<td>20</td>
</tr>
</tbody>
</table>

\[
\text{Ratio: } \frac{40}{30}, \frac{50}{30} = \frac{50}{20} \\
50 + 40x = 30 + 30(1-x) \\
50 + 40x = 67.14 \quad \text{split painting} \\
50 + 40x = 67.14 \\
70x = 3.75 \\
x = \frac{3.75}{70} \\
x = \frac{3}{40}
\]

7: Jack and Jill went up the hill and found an antique water pail. Jack and Jill must now make a fair division of the pail using the Knaster inheritance procedure. Jack bids $80 for the value of the pail and Jill bids $65. What is the outcome of the fair division?

a: Jack gets the pail and pays Jill $36.25
b: Jack gets the pail and pays Jill $40
c: Jack gets the pail and pays Jill $32.50
d: Jack gets the pail and pays Jill $65

8: A small county has populations in three districts as shown below. They are to apportion 10 seats on the county council. Find the quota for Riverdale.

<table>
<thead>
<tr>
<th>District</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parkview</td>
<td>43,000</td>
</tr>
<tr>
<td>Hillside</td>
<td>32,800</td>
</tr>
<tr>
<td>Riverdale</td>
<td>24,200</td>
</tr>
</tbody>
</table>

\[
\text{Average size} = \frac{100,000}{10} = 10,000 \\
\text{Riverdale quota} = \frac{24,200}{10,000} = 2.42
\]

9: We are scheduling seven course sections for a total of 217 students. Enrollments are: 109 in calculus 1, 79 in calculus 2, and 29 in advanced calculus. Find the apportionment for each course using the Jefferson method.

\[
\text{Tentative offer: calc 1} \quad \frac{109}{31} = 3.52 \quad 3 \\
\text{calc 2} \quad \frac{79}{31} = 2.55 \quad 2 \\
\text{adv calc} \quad \frac{29}{31} = .94 \quad 0
\]

\[
\frac{29}{29} = 29 \quad \text{give adv calc the first seat}
\]
10: In the following two-person game, the payoffs represent gains to the row Player 1 and losses to the column Player 2. Which of the following is true?

\[
\begin{bmatrix}
1 & 3 \\
4 & 2
\end{bmatrix}
\]

- The game has no saddle point
- The value 3 is a saddle point
- The value 2 is a saddle point
- The value 2.5 is a saddle point

20 pts. apiece

II: Free response:

1: Identify an Euler circuit on the graph below by numbering the sequence of edges in the order traveled.

There are many answers to this.

2: Find an eulerization with 9 added edges for a 3-by-6 block rectangular street network.

Almost Many answers

3: A vanity license can use any combination of numbers and letters from length 2 to length 4. How many possible vanity plates are there?

\[
36^2 + 36^3 + 36^4 + 36^5 + 36^6 + 36^7 = \frac{36^8 - 1}{36 - 1} - 37
\]

\[
36^2 + 36^3 + 36^4 + 36^5 = 62,193,744
\]
4: Given the order-requirement digraph below (with time given in minutes), apply the critical path scheduling algorithm to construct a schedule using three processors.

5: Kim and Lynn produce pottery vases and bowls. A vase requires 25 oz. of clay and 5 oz. of glaze. A bowl requires 20 oz. of clay and 10 oz. of glaze. There are 500 oz. of clay available and 160 oz. of glaze available. If the profit on one vase is $5 and the profit on one bowl is $3, how many of each should be made in order to maximize profit?

6: Apply the Northwest Corner Rule to the following tableau. Make two modifications to the distribution that leave the rim numbers fixed but decrease the cost (explain in one sentence what you did).
7: In recent labor-management negotiations, several issues were identified, and points assigned to them indicating relative importance to each side, as below. Use the adjusted winner procedure to determine a fair resolution between labor and management.

<table>
<thead>
<tr>
<th>Issue</th>
<th>Management's points</th>
<th>Labor's points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base pay</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>Incentive pay</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Health care benefits</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Worker safety</td>
<td>8</td>
<td>15</td>
</tr>
<tr>
<td>Opportunity for promotion</td>
<td>20</td>
<td>13</td>
</tr>
<tr>
<td>Retirement package</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Employee accountability</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Management gets Incentive pay, opportunity for promotion, employee accountability.
Labor gets Base pay, Health care benefits, worker safety.

\[46 + 20x = 50 + 20(1-x)\]
\[40x = 24\]
\[x = 0.6\]

Management controls 60% of retirement package while Labor controls 40%. Total points for each: 58

8: Four people must make a fair division of two classic cars. They use the Knaster inheritance procedure. Their bids are shown below. What is the division of the objects?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Bird</td>
<td>60,000</td>
<td>80,000</td>
<td>60,000</td>
<td>40,000</td>
</tr>
<tr>
<td>Mustang</td>
<td>80,000</td>
<td>60,000</td>
<td>80,000</td>
<td>100,000</td>
</tr>
</tbody>
</table>

B wins T-Bird, puts $60,000 in kitty.
A & C pull out 15k; D pulls out 10k, leaving 20k.
Split equally (5k apiece).

D wins Mustang, puts 75k in kitty.
A & C pull out 20k; B pulls out 15k, leaving 20k.
Split equally (5k apiece).

Upshot: A & C both get 45,000; B wins T-Bird, pays 35,000; D wins Mustang, pays 55,000.
9: Given the cities and the populations below, use the Hamilton method of apportionment to distribute the 25 seats on a regional board.

<table>
<thead>
<tr>
<th>City</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greenville</td>
<td>34,569</td>
</tr>
<tr>
<td>Riverdale</td>
<td>27,943</td>
</tr>
<tr>
<td>Oceanside</td>
<td>21,350</td>
</tr>
<tr>
<td>Parkview</td>
<td>16,138</td>
</tr>
</tbody>
</table>

\[
\text{Average size} = \frac{100,000}{25} = 4000
\]

Greenville & Riverdale get the extra seats since their fractional parts are the largest. Thus, Greenville gets 9 seats, Riverdale gets 7, Oceanside gets 5, and Parkview gets 4.

10: In the following two-person zero-sum game, the payoffs represent gains to the row Player I and losses to column Player II. Does this game have a saddlepoint? What is each player's minimax or maximin strategy?

\[
\begin{bmatrix}
3 & 6 & 7 \\
1 & 8 & 5 \\
4 & 2 & 9 \\
\end{bmatrix}
\]

\[
\begin{align*}
\text{Maximin} & = 3 \\
\text{Minimax} & = 2
\end{align*}
\]

Player 1 will choose the first row, Player 2 will choose the first column. 3 is not a saddlepoint since it is not the number at the bottom of column 1. 3 will be chosen.