8 pts.  
1: Calculate $5^{52}$ mod 21.

\[ 5^1 = 5; \quad 5^2 = 4 \pmod{21}; \quad 5^3 = 20 \pmod{21}; \quad 5^4 = 16 \pmod{21}, \]
\[ 5^5 = 17 \pmod{21}; \quad 5^6 = 1 \pmod{21}; \quad 5^{12} = 5 \pmod{21}, \]
\[ 5^{18} = 5 \pmod{21}; \quad 5^{24} = 1 \pmod{21}; \quad 5^{48} = 1 \pmod{21}, \]
\[ 5^{49} = 5 \pmod{21}; \quad 5^{50} = 4 \pmod{21}; \quad 5^{51} = 20 \pmod{21}, \]
\[ \boxed{5^{52} = 16 \pmod{21}} \]

12 pts.  
2: The Chemical Abstract Service uses the following scheme for registry numbers: $a_1a_2\ldots a_7$ are the identification digits, and the check digit is $a_8 = 7 \cdot a_1 + 6 \cdot a_2 + 5 \cdot a_3 + 4 \cdot a_4 + 3 \cdot a_5 + 2 \cdot a_6 + 1 \cdot a_7 \pmod{10}$. What is the check digit for the identification number 7352861? What single errors will not be detected? What transposition errors will not be detected? Explain your reasoning!

\[ a_8 = 7(7) + 6(3) + 5(5) + 4(2) + 3(8) + 2(6) + 1(1) \pmod{10} \]
\[ = 9 + 8 + 5 + 8 + 4 + 2 + 1 = 7 \pmod{10} \]

$a_8 = 7$ is the check digit.

- When we look at the rows for 2, 4, or 6 (mod 10), they have repeats in the $a_2 \rightarrow 5, a_5 \rightarrow 6, a_8 \rightarrow 7$ columns, so any errors like that will not be detected.
- When we look at the 5 row (mod 10), it goes 5050... so an even in this position is swapped for even, undetected. Same for odd-odd switches.
- All transpositions will be detected. Looking at 1st two positions, $7a_1a_2 = a_1a_2 \Rightarrow a_1 = a_2 \pmod{10}$ (only undetected when digits are the same, not really a transposition).