Portfolio Evaluation: Consider two stocks, \( a \) and \( b \), and let \( S_a(t) \) and \( S_b(t) \) be the prices of the two stocks at time \( t \). At time \( t = 0 \), you buy \( n_a \) shares of stock \( A \) and \( n_b \) shares of stock \( B \). Then your initial wealth is

\[
W_0 = n_a S_a(0) + n_b S_b(0).
\]

Suppose your investment horizon is \( T \in \mathbb{R} \) years, after which your terminal wealth \( W_T \) is given by

\[
W_T = n_a S_a(T) + n_b S_b(T).
\]

(This presumes you do not trade any of your stock in the time interval \([0, T]\).)

Assume that \( S_a \sim \text{GBM}(\mu_a, \sigma_a) \) and \( S_b \sim \text{GBM}(\mu_b, \sigma_b) \), where \( \text{GBM}(\mu, \sigma) \) corresponds to a Geometric Brownian Motion distribution given by the following equation:

\[
S_k(T) = S_k(0) \exp((\mu_k - \frac{\sigma_k^2}{2})T + \sigma_k B_k(T))
\]

and where \( B_k(T) \) is given by a Standard Brownian Motion distribution, i.e., a \( \text{Normal}(0, \sqrt{T}) \) distribution. Assume that \( B_a(T) \) and \( B_b(T) \) are independent.

You would like to estimate

\[
\Pr\left(\frac{W_T}{W_0} \leq 0.9\right),
\]

i.e., the probability that the value of your portfolio drops by more than 10%.

Let \( L \) be the (loss) event that \( \frac{W_T}{W_0} \leq 0.9 \). You can estimate the probability of \( L \) using the following characteristic function, where \( \mathbf{X} = (S_a(T), S_b(T)) \):

\[
I_L(\mathbf{X}) = \begin{cases} 1, & \text{if } \frac{n_a S_a(T) + n_b S_b(T)}{n_a S_a(0) + n_b S_b(0)} \leq 0.9 \\ 0, & \text{otherwise} \end{cases}
\]

computing \( N \) realizations of this characteristic function, and dividing the sum by \( N \):

\[
\hat{\theta}_N = \frac{I_L(\mathbf{X}_1) + I_L(\mathbf{X}_2) + \cdots + I_L(\mathbf{X}_N)}{N}
\]

Use Monte Carlo simulation to estimate this probability. Use the following parameter values:

- \( T = 0.5 \) years
- \( \mu_a = 0.15, \sigma_a = 0.20 \)
- \( \mu_b = 0.12, \sigma_b = 0.18 \)
- \( S_a(0) = $100, S_b(0) = $75 \)
- \( n_a = n_b = 100 \) shares

(Note that these parameter values give \( W_0 = $17,500 \).)

Implement a Monte Carlo simulation model for this portfolio using R. Experiment with various values for \( N \). Experiment with various values for \( T \), the terminal time in years. Generate some meaningful histograms. What kind of decisions can you make based on your results?

Submitting: Package your work (R source, appropriately labeled histograms as PNGs, a README containing appropriate discussion) into a gzipped tarball similar and drop your tarball into the shared Box folder for this class. Your lab is due by 23:59:59 on Sun 18 Feb.