Math 211
Test Two Nov 8, 2006
Name __________________________

As usual, do not use your book, notes, or calculator.

15. Each part takes just about a single step. Do NOT simplify answers.

\( y = \ln(\tan(x)) \)

\( y' = \frac{\tan(x)}{\cos^2(x)} \cdot \sec^2(x) \)

\( y = e^{-5x} \)

\( y'' = 25e^{-5x} \) Note: SECOND deriv here.

15. Get an equation involving \( y' \) for the graph of \( x - 3x + y + 5y = 6 \)
and then determine the equation of the tangent line to that graph at the
point \((0, 1)\). Show work clearly, using \( y' \) wherever appropriate.

\[ 2 \quad 2 \]
\[ \frac{\partial}{\partial y} \left( x - 3x + y + 5y = 6 \right) \]
\[ \frac{\partial}{\partial y} \left( x - 3x + y + 5y = 6 \right) \]
\[ (2y + 5)y' = -2 + 3x \]
\[ y' = \frac{-2 + 3x}{2y + 5} \]

Thus the tangent line goes through \((0, 1)\) and has slope \( \frac{3}{2} \).

So has equation \( y - 1 = \frac{3}{2} (x - 0) \)

\[ y = \frac{3}{2} x + 1 \]

Tangent line has equation \( y = \frac{3}{2} x + 1 \)

14. Yes or No? Put either Yes or No in each blank. (The letter \( f \) in any
of these problems is independent of the letter \( f \) in any other one of these)

Yes

Suppose a function \( f \) is differentiable at a number \( a \). Must \( f \) also be
continuous at \( a \)?

No

Suppose the graph of a function \( f \) is smooth when \( x = a \). Must that \( f \) be
differentiable when \( x = a \)?

Yes

Is there a function \( f \) and a number \( a \) such that \( f'(a) = 0 \) and also
such that \( f \) has an inflection point when \( x = a \)?

No

Must every max value for a function \( f \) also be a local max value for \( f \)?

Can a function have a local min where its derivative is positive?

Yes

Suppose \( f \) is differentiable on \([a, b]\). Must there be at least one number \( c \) in \([a, b]\) such that \( f(c) \) is as large as all other values of \( f(x) \) for \( x \) in \([a, b]\)? \( f \) is continuous on \([a, b]\), so apply Extreme Value Theorem.

Yes

Suppose \( f \) is continuous on \([a, b]\). Must there be at least one number \( c \) in \([a, b]\) such that \( f(c) \) is as large as all other values of \( f(x) \) for \( x \) in \([a, b]\)?

By Extreme Value Theorem

Do the remaining parts of this problem after finishing Problems

No

Suppose \( f \) is continuous on \([a, b]\). Must there be at least one number \( c \) in \([a, b]\) such that \( f(c) \) is as large as all other values of \( f(x) \) for \( x \) in \([a, b]\)?

Consider \( f(x) = \frac{x}{x - 1} \) with \( a = 0 \) and \( b = 1 \)

No

Suppose \( f \) is differentiable on \([a, b]\). Must there be at least one number \( c \) in \([a, b]\) such that \( f'(c) = 0 \)?

Consider \( f(x) = x \)

Suppose \( f'(x) \) is positive when \( x = c \). Must \( f \) be increasing when \( x = c \)?

No

Suppose \( f'(x) \) is positive when \( x = c \). Must \( f \) be increasing when \( x = c \)?

Consider \( f(x) = e^x \)

No

Suppose \( f' \) is positive when \( x = c \). Must \( f' \) be positive when \( x = c \)?

Consider \( f(x) = x^3 \) and \( c = 0 \)

Yes

Consider the function \( f(x) = x \). Does the Mean Value Theorem guarantee
that there is a number \( c \) in \([0, 2]\) such that \( f'(c) = \frac{1}{2} \)?

\[ f'(x) = \frac{2}{x} \]

Note that \( f'(x) = \frac{2}{x} \) when \( x = 0 \)

No

Consider the function \( f(x) = x^3 \). Does the Mean Value Theorem guarantee
that there is a number \( c \) in \([-1, 1]\) such that \( f'(c) = 1 \)?

\[ f'(x) = 3x^2 \]

Note that \( f'(x) = 3x^2 \) when \( x = 0 \)

No

Consider the function \( f(x) = x^3 \). Does the Mean Value Theorem guarantee
that there is a number \( c \) in \([-1, 1]\) such that \( f'(c) = -1 \)?

\[ f'(x) = 3x^2 \]

Note that \( f'(x) = 3x^2 \) when \( x = 0 \)

No

Consider the function \( f(x) = x^3 \). Does the Mean Value Theorem guarantee
that there is a number \( c \) in \([-1, 1]\) such that \( f'(c) = 1 \)?

\[ f'(x) = 3x^2 \]

Note that \( f'(x) = 3x^2 \) when \( x = 0 \)

Yes

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Yes

Consider the function \( f(x) = x^3 \). Does the Mean Value Theorem guarantee
that there is a number \( c \) in \([-1, 1]\) such that \( f'(c) = 1 \)?

\[ f'(x) = 3x^2 \]

Note that \( f'(x) = 3x^2 \) when \( x = 0 \)

Yes
6. Consider a function \( f \) all of whose points lie above the \( x \)-axis such that \( f \) has each of the following 10 additional properties:

1) \( f'(x) < 0 \) for all \( x \) in the interval \(( -\infty, 1)\)
2) \( f'(1) = 0 \)
3) \( f'(x) < 0 \) for all \( x \) in the interval \(( 1, 4)\)
4) \( f \) is continuous at \( 4 \)
5) \( f'(4) \) does not exist
6) \( f''(x) > 0 \) for all \( x \) in the interval \(( 4, +\infty)\)
7) \( f''(x) < 0 \) for all \( x \) in the interval \(( -\infty, 1)\)
8) \( f''(x) < 0 \) for all \( x \) in the interval \(( 1, 3)\)
9) \( f''(x) > 0 \) for all \( x \) in the interval \(( 3, 4)\)
10) \( f''(x) < 0 \) for all \( x \) in the interval \(( 4, +\infty)\).

Based solely on the information provided above about \( f \), answer parts (a) through (d).

(a) At what value of \( x \) in the interval \([ 1, 3] \) must \( f \) have its smallest value?
Answer: when \( x = 3 \)

(b) \( f \) has an inflection point when \( x = \frac{1}{3}, 4 \) (give all answers, if there are any answers).

(c) Critical numbers occur when \( x = 1, 4 \) (give all answers, if there are any answers).

(d) Is it possible for an \( f \) satisfying the given conditions to have an absolute maximum value?
- yes  \( \checkmark \)  no  \( \not\checkmark \)  Follows from properties (6) and (7)

(e) Graph \( f \). IF YOUR GRAPH APPEARS TO HAVE BENDS THAT CONTRADICT THE ABOVE INFO, YOU CAN LOSE ALL 10 POINTS. So draw your graph precisely and neatly. Here’s how you should proceed:
First sketch your graph in the scratch work area on the right. Make sure all points of \( f \) lie above the \( x \)-axis. Then systematically check each of the given 10 properties of \( f \) for your tentative sketch. Only then should you copy your graph below.
Each of the stated properties is worth 1 point.

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IF TIME REMAINS, BE SURE TO FINISH PROBLEM 4.

PLEDGE: On my honor I have neither given nor received unauthorized help on this test.