SPECTRAL DECOMPOSITION TECHNIQUES FOR THE INCOMPRESSIBLE NAVIER-STOKES EQUATION

Joel Avrin, Mathematics, University of North Carolina-Charlotte

Spectral decompositition techniques that we have found useful in obtaining large-frequency global regularity and one-point-attractor results for the incompressible Navier-Stokes equations with (at least partially) zero boundary conditions can be adapted to improve the global regularity results of Raugel/Sell and Moise/Temam/Ziane. In contrast to these works we use a purely semigroup approach.

USE OF A REDUCED ORDER SURFACE KINETICS MODEL AND P-POLARIZED REFLECTANCE MEASUREMENTS FOR REPRESENTATION OF GAP FORMATION

Scott Beeler, Department of Mathematics, North Carolina State University
(*joint work with H. T. Tran and N. Dietz*)

This presentation will discuss a reduced order surface kinetics (ROSK) model which has been developed to describe the chemical vapor deposition method of thin film growth with a system of ordinary differential equations. The model describes the decomposition kinetics of the precursors and their incorporation into the film. Real-time measurements made by p-polarized reflectance spectroscopy (PRS) during GaP growth are used to follow changes in the composition and thickness of the surface reaction layer during the periodic precursor cycle. Linkage of the PRS response to the ROSK model provides the means for parameter estimation of rate constants and optical response factors, and also gives insights into the precursor decomposition and growth kinetics.

EXACT CONTROLLABILITY OF A HANGING CABLE

Boris P. Belinskiy, Department of Mathematics, University of Tennessee at Chattanooga
(*joint work with Marianna A. Shubov, Texas Technical University*)

We study the controllability problem for a non-homogenies non-extendable hanging cable with no resistance to banding. The cable is suspended at one end and carrying a mass at the other end. The (concentrated) force is acting on the mass and is considered as a control. This model was studied by Woodward, who considered only homogenies cable fixed at one end. He derived the equation for eigenfrequencies in terms of Bessel functions and calculated numerically some first eigenfrequencies. We consider a non-homogeneis cable under the action of a concentrated force applied to the mass in the direction perpendicular to the cable. We treat this force as a control. The multi- component structure of the model allows us to control it by using the concentrated force. Generalizing the classical results byD.L. Russell, we give necessary and sufficient conditions for exact controllability of the system. We use our recent results concerning the spectral analysis of a class of non- selfadjoint operators and operator pencils generated by the model above. The paper represents the continuation of our previous results on exact controllability of the damped wave equation.

INTEGRALS OF LEGENDRE POLYNOMIALS AND SOLUTION OF SOME PARTIAL DIFFERENTIAL EQUATIONS

Rachel Belinsky, Mathematics, Morris Brown College

Integrals of Legendre Polynomials do not belong to the family of classical orthogonal polynomials. In my talk I'll show some properties of these polynomials. Then I'll use them to solve some partial differential equations. In addition, I'll show new formulas for some classical orthogonal polynomials.

SUMMATION OF POLYPARAMETRICAL FUNCTIONAL SERIES BY THE METHOD OF FINITE HYBRID INTEGRAL TRANSFORMS (HANKEL 1, FOURIER)

Andriy Blazhievskiy , Mathematics & CS, Technological University of Podillia

The design of physical and mechanical characteristics of thin isotropic nonhomogeneous (partly-homogeneous) plates of limited size according to the degree low lead to the construction of solution for the separate system of differential equations on the given initial conditions, boundary conditions and the conditions of thermo-mechanical contact in the connecting planes. Let one end of the plates is under the actions of the spasmodic heat regime, or the heat sources act on the plane part accordingto the spasmodic law. The stationary state of system is described by the functions depending on functional series consisting of the combination of the trigonometric and Bessel functions. Since we deal easier with functions than with series we encounter with the problem of function series summation. The article is devoted to the summation of just such series by the method of finite hybrid integral transforms Hankel 1-Fourier.

By the Cauchy's method for the separate system of the ordinary differential equations we have constructed the solution of the corresponding boundary problem in the case of general assumption on the differential and connected operators. The condition of the nonlimited solving and the structure of the general solution for the boundary problem have been written in the explicit form. On the other hand solution of this problem has been constructed by the method of the finite hybrid integral transforms. Since this problem has one and only one solution we may compare the first solution with the second and, as a result, get the sums of functional series.

COMPUTATIONAL INVERSE EIGENVALUE PROBLEM

Edward Kwasi Boamah, Computational and Applied Mathematics, University of the Witwatersrand, Johannesburg

This research work is concerned with the Computational Inverse Eigenvalue Problem of the Sturm-Liouville differential equation. The central problem considered is the approximate reconstruction of an unknown coefficient function in the Sturm-Liouville Differential Equation from a given finite spectral data set. A numerical solution will be sought using a finite element discretization method. Numerical results will be presented to illustrate the effectiveness of the method and Matlab programming software used.

CONTINUOUS REARRANGEMENT AND SYMMETRY OF SOLUTIONS OF BVP'S

Friedemann Brock, Mathematics, University of Missouri-Columbia

Let $u \in W_0^{1,p}(B) \cap C^1(\overline{B})$ satisfy $u \geq 0$ and $\nabla(|\nabla u|^{p-2}\nabla u) + f(u) = 0$ in B, where $p > 1$, f is continuous and B is a ball in $\mathbf{R}^n$. Using the method of continuous Steiner symmetrization we prove that u has certain 'local' symmetry properties. We show furthermore that u is radially symmetric and radially decreasing in some special cases, e.g. if f is positive.

GLOBAL SOLUTIONS FOR A CLASS OF DISPERSIVE EQUATIONS

Radu C. Cascaval, Mathematical Sciences, University of Memphis
(*joint work with Jerome A. Goldstein, University of Memphis*)

We present a semigroup approach for the existence theory of the class of nonlinear dispersive equations of the form:

$$u_t - Lu_x + F(u)_x = 0,$$

where L is a pseudo-differential operator of order $\beta \geq 1$ and F is a nonlinear function satisfying a certain growth condition at infinity. This class includes the generalized Korteweg-de Vries equations and the Benjamin-Ono equation. The growth of different Sobolev norms of solutions are estimated and the continuous dependence on initial data is discussed.

THE EXISTENCE AND BEHAVIOR OF SINGULARITIES OF SOLUTIONS TO THE FLOW OF H-SYSTEMS

Stacey Chastain, Mathematics, University of Florida
(*joint work with Yunmei Chen*)

We prove the existence of a unique, regular solution to the flow of the H-system,

$$u_t - \Delta u = 2H(u)u_x \wedge u_y$$

with Dirichlet boundary condition, where $H : R^3 \to R$ is a Lipschitz continuous function. The solution exists at least up until the time of an energy concentration. Furthermore, if the solution satisfies a certain energy inequality, then it can be extended to a global weak solution which is regular with the exception of finitely many singularities. The behavior of the solution at these singularities will also be discussed.

BLOW-UP ESTIMATES OF POSITIVE SOLUTIONS OF A REACTION- DIFFUSION SYSTEM

Hongwei Chen, Department of Mathematics, Christopher Newport University

This paper is concerned with positive solutions of the reaction-diffusion system

$$u_t - \Delta u = u^{m_1} v^{n_1},$$

$$v_t - \Delta v = u^{m_2} v^{n_2},$$

which blow up at $t = T$. The conditions on exponents which yield the blow-up rate estimates are obtained and a complete classification of blow-up patterns is derived.

ELECTRICAL IMPEDANCE TOMOGRAPHY

Margaret Cheney, Mathematical Sciences, Rensselaer Polytechnic Institute

Electrical impedance tomography systems are being developed for medical diagnostics, detection of cracks in metals, and imaging of multiphase flow. Such systems work by applying currents to electrodes at the surface of the body, measuring the resulting voltages, and reconstructing the electrical impedance in the interior of the body. This talk will focus on various problems arising in the design of such systems and in the processing of the data they measure.

RECENT DEVELOPMENTS ON SEMIPOSITONE SYSTEMS

Maya Chhetri, Mathematical Sciences, University of North Carolina at Greensboro
(*joint work with Alfonso Castro and Ratnasingham Shivaji*)

We present the very recent results on semipositone systems. In particular, we discuss the positivity result and an existence result for classes of cooperative elliptic semipositone systems and an existence result for a class of quasilinear semipositone systems. We use reflection arguments, sub-super solution method and degree theory to prove these results. We also discuss some open problems.

PERIODIC SOLUTIONS OF NON HOMOGENEOUS DIFFERENTIAL EQUATIONS

A. Chouikha, Mathematics, University of Paris-Nord

We study the period function $T(c)$ of a center of non homogeneous differential system: (1) $u''+g(t,u,u')=0$ where the function g satisfies certain conditions, ensuring the existence of a center at the origin O.

We are interested in the existence of solutions of equation (1), and more precisely in the case where $g(t,u,u') = f(u) + \epsilon h(t,u,u')$. So that the homogeneous equation $u'' + f(u)$ has a center and a monotonic period function depanding on the energy. Some sufficient conditions for the monotonicity of $T(c)$ or for the isochronocity of a center in the homogeneous case may be extend in the perturbed case.

We also examine the differential equation of Lienard type $u'' + a(t)u' + f(u) = 0$, and the monotonicity of its period function.

A MATHEMATICAL MODEL FOR THE UPTAKE AND ELIMINATION OF BENZENE IN ANIMALS

Cammey Cole, Dept. of Mathematics, Center for Research in Scientific Computation, North Carolina State University
(*joint work with Paul M. Schlosser and Hien T. Tran*)

Benzene is a known human leukemogen found in industrial emissions as well as in cigarette smoke and automobile emissions. The exposure-dose-response relationship of inhaled chemicals such as benzene is complicated because many of the toxic effects occur in tissues other than the lung, which is the site of absorption. In this presentation, we will describe a physiologically based pharmacokinetic model for the uptake and elimination of benzene in animals. This model will be used to relate the concentration of inhaled benzene to tissue dose. Various inverse problem formulations, including weighted least-squares and maximum likelihood functions, have been implemented to determine model parameters that are not available in the literature.

OSCILLATORY THEOREMS OF DELAY AND ADVANCED DIFFERENTIAL EQUATIONS

Raj Dahiya, Department of Mathematics, Iowa State University

Abstract forthcoming

NONLINEAR EIGENVALUE PROBLEMS INVOLVING TWO CLASSES OF FUNCTIONAL DIFFERENTIAL EQUATIONS

John Davis, Dept of Mathematics, Baylor University
(*joint work with K.R. Prasad and W.K.C. Yin*)

We consider the eigenvalue problem $(-1)^{n-k}u^{(n)}(t) = \lambda a(t)f(u_t)$, $0 \leq t \leq 1$, satisfying either conjugate or right focal boundary conditions plus an initial condition. Values of the parameter λ (eigenvalues) are determined for which this problem has a positive solution. The methods used here extend recent works by allowing for a much broader class of functions for $a(t)$ and by providing optimal eigenvalue intervals.

PERSISTENCE CRITERIA FOR A VARIABLE NUTRIENT CHEMOSTAT

Sean Ellermeyer, Mathematics, Kennesaw State University
(*joint work with S.S. Pilyugin*)

In this work, we modify the standard single species chemostat model to include a variable nutrient input which is assumed only to be nonnegative, bounded, and continuous. We obtain necessary and sufficient conditions for persistence in terms of the nutrient input and the other relevant parameters of the chemostat. In particular, we introduce the ideas of strong and weak "average" persistence and we show that there exist cases in which these types of persistence are not equivalent to the usual concepts of strong and weak persistence. We also show that the persistence type of each single species model depends only on the parameters of the model and not on particular solutions.

OPTIMAL CONTROL OF HARVESTING COUPLED WITH BOUNDARY CONTROL IN A PREDATOR-PREY SYSTEM

K. Renee Fister, Mathematics and Statistics, Murray State University

We consider boundary control and control via harvesting in a parabolic predator-prey system for a bounded region. The boundary control depicts the relationship between the boundary environment and the possibly harmful species. In addition, a proportion of the predator is harvested for profit. We choose to maximize the objective functional which incorporates the size of the prey and the revenue of harvesting of the predator less the economic cost of sustaining a satisfactory boundary habitat and the cost due to the harvesting component. Moreover, we characterize the unique optimal control in terms of the solution of the optimality system, which is the state system coupled with the adjoint system.

PERMANENCE IN THE MEAN FOR STOCHASTIC POPULATION MODELS

Tom Gard, Mathematics, University of Georgia

Asymptotic moment bounds for stochastic population models are obtained by applying a theorem of Miyahara on ultimate boundedness of solutions of stochastic differential equations.

A NEW, EXPONENTIALLY ACCURATE SEMICLASSICAL APPROXIMATION

George A. Hagedorn, Department of Mathematics, Virginia Tech
(*joint work with Alain Joye*)

We present results on the semiclassical limit of quantum mechanics that have been obtained recently in collaboration with Alain Joye.

We derive a new and relatively simple asymptotic expansion in powers of $\hbar^{1/2}$ for certain solutions to the d–dimensional time–dependent Schrödinger equation. By optimal truncation of this expansion we obtain approximations that agree with exact solutions up to errors whose norms are bounded by $C\ \exp\left(\gamma/\hbar\right)$, where C and γ are positive. Depending on the hypotheses, these results are proved for compact time intervals or time intervals that grow like $\log(\hbar)$. In the context of scattering theory, we obtain results for infinite times.

SYMBOLIC COMPUTATION OF THE LIE POINT SYMMETRIES OF THE VAIDYA EQUATION

Russell Herman, Mathematics and Statistics, UNC Wilmington

We present the symmetries of one of the Vaidya equations, $\gamma\gamma_u + \gamma_\alpha - \gamma\cot\alpha - i\gamma_\beta\csc\alpha = 0$, which arises in the study of the gravitational field in the exterior of a radiating, spherically symmetric star. The determination of symmetries of nonlinear partial differential equations typically leads to the need to integrate a large system of determining equations for the infinitesimals of the group. In recent years algorithms, which reduce these systems of overdetermined PDEs into a standard involutive form, have been introduced and automated using a variety of symbolic software. In this talk we will present the fifteen Lie Point symmetries of the Vaidya equation, which we have found, and some of the problems faced in using typical symbolic packages.

BOUNDARY STABILIZATION OF A SYSTEM OF ANISOTROPIC ELASTICITY WITH LIGHT INTERNAL DAMPING

Mary Ann Horn, Department of Mathematics, Vanderbilt University

Controllability and stabilization of more complex, connected systems becomes more and more of an issue as results on single components become more numerous. An anisotropic system of elasticity is considered containing light internal damping which is physically motivated. To uniformly stabilize the system, boundary damping is introduced, acting via traction forces. With this additional damping, exponential decay of the energy with respect to time is established. Connections to the system of linear elasticity and extensions to more complex systems will also be discussed.

TIME-DEPENDENT PERTURBATION AND THE BORN-OPPENHEIMER APPROXIMATION

Steven Jilcott, Mathematics, Virginia Tech

We discuss the physical problem of a molecule interacting with an electromagnetic field pulse and model the problem using a time-dependent perturbation of the Born-Oppenheimer approximation to the Schrödinger equation. Using previous results that develop asymptotic series solutions in the Born-Oppenheimer parameter ϵ, we derive a formal Dyson series expansion in the perturbation parameter μ. We then prove that this series is asymptotically accurate in both parameters, provided that the Hamiltonian for the electrons has purely discrete spectrum.

SMOOTHNESS OF SOLUTIONS OF CONJUGATE BOUNDARY VALUE PROBLEMS

Eric R. Kaufmann, Mathematics and Statistics, University of Arkansas at Little Rock

In this paper we consider the n^{th} order Δ-differential equation (often referred to as a differential equation on a measure chain) $u^{\Delta_n}(t) = f(t, u(\sigma(t)), \ldots, u^{\Delta_{n-1}}(\sigma(t)))$ satisfying n-point conjugate boundary conditions. We show that solutions depend continuously and smoothly on the boundary values.

PRESCRIBING HIGHER ORDER CURVATURE IN CONFORMAL DIFFERENTIAL GEOMETRY

Michael K.-H. Kiessling, Mathematics, Rutgers University

In recent years there has been an increasing interest in Paneitz' 4-th order PDE on 4-manifolds, and it's n-th order generalizations on n-dimensional manifolds, which are for n-manifolds what Liouville's equation is for the problem of constructing a surface conformally equivalent to a given surface, having prescribed Gauss curvature, a.k.a. Nirenberg's problem. In this talk we show that a wide class of such n-manifolds with prescribed "Paneitz-curvature" can be constructed with a quite universal technique developed originally for the statistical mechanics of fluid vortices.

MIXED ORDER SYSTEMS OF ORDINARY LINEAR DIFFERENTIAL EQUATIONS

Siaka Kone, Mathematics, University of the Witwatersrand , Johannesburg, South Africa

We have characterized a sufficiently large classes of mixed order systems of differential equations which are equivalent to first order systems. This has been shown through the reduction of mixed order systems to first order systems. Writing mixed order systems in operator form, we have shown that this operator is equivalent to an operator for associated first order systems.

A DISPERSIVE DETERMINISTIC ANALOGUE OF TURBULENCE

Peter Lax, Courant Institute of Mathematical Sciences, New York University

There is a large class of equations containing a small parameter whose solution as the parameter tends to zero converges only weakly in certain portions of space-time. Because of nonlinearity these weak limits do not satisfy the equation obtained by setting the parameter equal zero. Examples where the limits can be described are the Korteveg-de Vries equation, the classical limit of the cubic Schroedinger equation, and the Toda lattice. The presentation will be expository.

A PARTIAL DETERMINATION OF $\partial\Omega$ BY THE HIGH ENERGY SPECTRUM OF THE DIRICHLET TO NEUMANN OPERATOR

Carl Lutzer, Mathematics, University of Kentucky

This talk will present results obtained with P. Hislop regarding the Dirichlet to Neumann operator, Λ, on bounded, connected domains $\Omega \subset R^n$ with smooth boundary. In particular, a lower bound on the number of holes in Ω will be determined by the asymptotics of the spectrum by demonstrating the diagonalization of Λ at high energy. Further, we will discuss the localization near $\partial\Omega$ of the harmonic extensions of eigenfunctions of Λ (in an L^2 sense) and conditions under which this localization happens at an exponential rate (as the eigenvalue tends to infinity).

ON THE OPTIMAL DESIGN OF TUBULAR STRUCTURES

C. Maeve McCarthy, Mathematics & Statistics, Murray State University

Given a fixed volume of material and the elastic properties of that material, the optimal design of the tallest tubular column under selfweight is sought. This problem can be formulated as an extremal eigenvalue problem. Considering columns of annular cross-section, the spectrum associated with physical designs is characterized. Existence of an optimal design is proven through the use of rearrangements. Necessary conditions of optimality are established.

SCALAR INTERMITTENCY IN SHEAR MODELS AND THE RIGOROUS CALCULATION OF A PDF TAIL

Richard M. McLaughlin, Mathematics, UNC Chapel Hill
(*joint work with Jared C. Bronski, Univ. of Illinois, Urbana*)

It is a well documented, experimental fact that while single point velocity measurements in turbulent fluids admit nearly Gaussian statistics, measurements of other quantities including velocity increments, vorticity, pressure, and passively transported quantities admit strongly non-Gaussian statistics. There has been an intense and important phenomenological effort attempting to explain this behavior in the context of a passive scalar using a variety of closure approximations and other ad hoc approximations. Even in the context of a passive scalar, the question of inherited statistics is extremely difficult, requiring the consideration of partial differential equations with variable, random coefficients.

In this lecture, we study the model introduced by Majda which concerns a passive scalar decaying in the presence of a rapidly fluctuating, Gaussian linear shear profile. We present the first explicit construction of the limiting large statistical moment asymptotics for the normalized scalar, and use this information to rigorously deduce the pdf tail. This elementary field theory shows the pdf tail ranging from Gaussian through stretched exponential as a parameter is varied, and we additionally obtain an explicit relation between the tail of the scalar and the tail of the scalar gradient which explicitly demonstrates how derivatives may become more intermittent.

A NONSTANDARD FINITE DIFFERENCE SCHEME FOR A STRONGLY NONLINEAR DUFFING EQUATIONS

Ronald E. Mickens, Physics, Clark Atlanta University

We consider a strongly nonlinear Duffing's ODE having no linear term in its restoring force. From the first-integral, it follows that all its solutions are periodic. From rules for nonstandard finite difference scheme [1] and exponetial integrators for differential equations [2], we construct a discrete model for the Duffing equation and we use it to provide numerical solutions. The mathematical and numerical properties of this scheme are discussed.

* Supported in part by research grants from DOE and the MBRS Program at Clark Atlanta University.

References

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THE TRANSMISSION PROBLEM FOR MULTILAYERED ANISOTROPIC ELASTIC BODIES WITH ROUGH INTERFACES

Dorina Mitrea, Mathematics, University of Missouri-Columbia

The general transmission problem of steady state oscillations for multilayered homogeneous anisotropic elastic bodies with Lipschitz interfaces is solved. We employ boundary integral methods, the solution being expressed in terms of layer potentials. The transmission problem we solve arises in solid mechanics and geophysical sciences.

ON THE SPECTRA OF ELASTOSTATIC AND HYDROSTATIC LAYER POTENTIALS ON CURVILINEAR POLYGONS.

Irina Mitrea, School of Mathematics, University of Minnesota

We prove that the spectral radii of certain elastostatic and hydrostatic boundary layer potentials in L^p, $2 \leq p < \infty$, on bounded curvilinear polygons are less than one. Such results are relevant in the context of constructively solving boundary value problems.

PERTURBATION METHODS APPLIED TO JOSEPHSON JUNCTIONS

Ben Rhouma Mohamed, Center for Dynamical Systems, CDSNS- Georgia Tech
(*joint work with Ben Rhouma Mohamed-Carmen Chicone*)

We study the persistence of periodic orbits for weakly coupled oscillators in general. We derive the necessary conditions for the survival of periodic solutions and apply the results to weakly coupled Josephson junctions previously studied by Aronson and Hadley. In particular we study the pure resitive case and the pure capacitive case and show that for both situations many interesting running solutions survive. Moreover as the bias current is either sufficiently small or sufficiently large we prove that there can be as many as 2^n running solutions where n is the number of junctions present in the system.

USE OF THE RESIDUAL AS AN ERROR ESTIMATOR FOR FREDHOLM AND HAMMERSTEIN EQUATIONS

Richard Noren, Mathematics and Statistics, Old Dominion University
(*joint work with Richard Noren and Hideaki Kaneko*)

We show that the residual can be used to estimate the error of a numerical solution for a class of nonlinear Hammerstein equations. It is also shown that the superconvergence of the iterated numerical solution provides a sufficient condition for the residual to be used as an error estimator. Hammerstein equations with smooth as well as weakly singular kernels will be treated.

A MODEL FOR THE BIOREMEDIATION OF A RIVER

Seth F. Oppenheimer, Mathematics and Statistics, Mississippi State University
(*joint work with Paul Waltman*)

We will consider the problem of the bioremediation of a river. We model introducing "pollution eating" organisms at a point source of pollution. The river will be modeled two ways; a half line (a really long river) and an interval. In both cases we impose zero right hand Dirichlet conditions, modeling a flow into a clean body of water. The left hand boundary condition will be a forced flux condition which also allows for flux due to convection–a Danckwerts condition. This model can also be viewed as a generalization of the unstirred chemostat problem. This is joint work with Paul Waltman of Emory University.

ABSTRACT EXTINCTION PROBLEMS

Mark E. Oxley, Mathematics and Statistics, Air Force Institute of Technology

Let X be a Banach space and C be an ordering cone in X. Let A be the generator of a C_0 semigroup $\{S(t) : t > 0\}$. We pose an (abstract) extinction problem as follows. Let $0 \neq z \in C$ and assume the function $u(t) = S(t)z$ is a strong solution to the initial-value problem

$$u_t = A\,u\ ,\quad u(0) = z\ .$$

We say u extincts in finite time if there exists a time $T \in (0, \infty)$ such that $u(t) = 0$ for all $t \geq T$. We discuss necessary and sufficient conditions on the generator A such that u extincts in finite time.

AN AVERAGING METHOD FOR A "TRULY" NONLINEAR CLASS OF OSCILLATOR ODE'S*

'Kale Oyedeji, Physics, Morehouse College

We study nonlinear oscillators containing a small parameter, ϵ. These oscillators are 'truly" nonlinear in the sense that for $\epsilon = 0$ the resulting ODE is nonlinear. Our main result is that the method of averaging (slowly varying amplitude and phase [1]) can be generialized and applied to this class of equations [2] . The procedure is illustrated by applying it to a modified van der Pol equation.

Research supported by a Morehouse College Faculty Research Grant.

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EQUIVALENCE BETWEEN REGULARITY THEOREMS AND HEAT KERNEL ESTIMATES FOR HIGHER ORDER ELLIPTIC OPERATORS UNDER DIVERGENCE FORM.

M. Qafsaoui, france, lamfa - universit de picardie - france
(*joint work with P. Auscher*)

We study the heat kernel of higher order elliptic operators under divergence form on $\mathbf{R}^n$. Ellipticity is in the sense of Garding inequality. We show that for homogeneous operators Gaussian upper bounds and Holder regularity of the heat kernel is equivalent to local regularity of weak solutions. Such a criterion allows to obtain heat kernel bounds for operators with uniformly continuous or vmo coefficients.

INVERSE PROBLEMS FOR ELASTIC MEDIA

Lizabeth Rachele, Mathematics, Tufts University

We consider the unique determination of material properties of bounded, nonhomogeneous, anisotropic elastic objects from measurements made at the surface.

The behavior of the object is modeled by solutions of hyperbolic systems of equations for anisotropic elastodynamics. Surface measurements are modeled by the Dirichlet-to-Neumann map on a finite time interval.

ASYMPTOTIC STABILITY IN LINEAR VOLTERRA DIFFERENCE EQUATIONS

Youssef N. Raffoul, Mathematics, University of Dayton

We use the resolvent matrix solution and Lyapunov functionals to obtain necessary and sufficient conditions for the stability the zero solution of linear Volterra difference equations.

NONEXISTENCE OF NONNEGATIVE SOLUTIONS FOR A CLASS OF SEMILINEAR ELLIPTIC SYSTEMS

Ratnasingham Shivaji, Mathematics and Computer Science, Georgia Southern University & Mississippi State University
(*joint work with Hai Dang*)

We consider classical nonnegative solutions ($u \geq 0, v \geq 0$) for the system

$$\begin{aligned} -\Delta u(x) &= f(v); \quad x \in \Omega \\ -\Delta v(x) &= g(u); \quad x \in \Omega \\ u = 0 &= v; \quad x \in \partial\Omega \end{aligned}$$

where Ω is either a ball or an annulus in $\mathbf{R}^n$; $n > 1$, $\partial\Omega$ being boundary of the ball. Here $f, g : [0, \infty) \to \mathbf{R}$ are C^1 functions satisfying

$$f(0) < 0 \quad \text{and} \quad \mathrm{g}(0) < 0, \quad \text{(semipositone system)}$$

and

$$f' \geq 0 \quad \text{and} \quad \mathrm{g}' \geq 0, \quad \text{(cooperative system)}$$

and there exist $K_i \geq 0$, $M_i \geq 0$; $i = 1, 2$ such that $\forall z \geq 0$

$$f(z) \geq K_1 z - M_1 \quad \text{and} \quad \mathrm{g(z) \geq K_2 z - M_2}.$$

We prove that the problem has no nonnegative solution for λ large.

ON DIFFUSION-INDUCED GRAIN-BOUNDARY MOTION

Gieri Simonett, Department of Mathematics, Vanderbilt University
(*joint work with U.F. Mayer*)

We consider a model which describes diffusion-induced grain-boundary motion of a surface which separates different grains in a poly-crystalline material. This model leads to a fully nonlinear coupled system of parabolic differential equations where the unknowns are the concentration of solute atoms within the moving interface and the position of the moving interface. We provide existence and uniqueness of smooth solutions using optimal regularity results.

LOCALIZATION IN ONE DIMENSIONAL RANDOM MEDIA: A SCATTERING THEORETIC APPROACH

Robert Sims, Mathematics Department, University of Alabama at Birmingham
(*joint work with Günter Stolz*)

We use scattering theoretic methods to prove exponential localization for random displacement models in one dimension. The operators we consider model both quantum and classical wave propagation. Our main tools are the reflection and transmission coefficients for compactly supported single site perturbations. We show that randomly displaced, non-reflectionless single sites lead to localization.

EXISTENCE OF NONOSCILLATORY SOLUTIONS FOR NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS OF SECOND ORDER

Bhagat Singh, Dept. Mathematics, University of Wisconsin - Manitowoc
(*joint work with Takasi Kusano*)

In this paper we are concerned with the nonlinear integro-differential equation (A) $y''(t)+a(t)\,|y(t)|^{\alpha-1}\,y(t)+\int_t^\infty b(t,s)\,|y(s)|^{\beta-1}\,y(s)\;ds=0,\quad t\geq 0$, where α and β are positive constants and $a:[0,\infty]\rightarrow(0,\infty)$ and $b:\Delta\rightarrow(0,\infty)$ are continuous functions, $\Delta=\{(t,s)\in\mathcal{R}^2 : s\geq t, t\geq 0\}$. We assume that $\int_t^\infty b(t,s)\,ds<\infty$ for all t ≥ 0. Our main purpose here is to characterize the existence of nonoscillatory solutions of equation A.

PERIODIC SOLUTIONS OF NONLINEAR WAVE EQUATIONS WHEN THE RATIO OF THE PERIOD TO THE LENGTH OF THE INTERVAL IS IRRATIONAL

Zachariah Sinkala, Mathematical Sciences, Middle Tennessee State University

In this paper, we consider the problem of finding a nonconstant, time periodic solution $u=u(x,t), 0\leq x\leq \pi, t\in\Re$, of the problem

$$u_{tt}-u_{xx}=-u\mid u\mid^{p-2}\quad(0,\pi)\times\Re,$$

$$u(0,.)=u(\pi,0)=0,$$

$$u(.,t+T)=u(.,t)\;\; for\ all\ t\in\Re,$$

where $p>2$ and the period T is given such that $\frac{T}{2\pi}$ is an irrational number having a bounded continued fraction, the above equation has a solution. The results in this paper extend some results of Rabinowitz.

ON THE CONNECTEDNESS OF THE SPACE OF INITIAL DATA FOR THE EINSTEIN EQUATIONS

Brian Smith, mathematics, University of Alabama at Birmingham
(*joint work with Gilbert Weinstein*)

Is the space of initial data for the Einstein vacuum equations connected? As a partial answer to this question, we prove the following result: Let $\mathcal{M}$ be the space of asymptotically flat metrics of non-negative scalar curvature on $\mathbf{R}^3$ which admit a global foliation outside a point by 2-spheres of positive mean and Gauss curvatures. Then $\mathcal{M}$ is connected.

REGULARITY OF SOLUTIONS TO THE PERTURBED CONSERVATION LAWS

Yudi Soeharyadi, Mathematical Sciences, The University of Memphis
(*joint work with Jerome A. Goldstein*)

A kind of regularity for the mild solution of perturbed conservation laws is proposed. This regularity is described in term of variations measured in the L^1-norm. A dissipativity condition from the semigroup approach is used to show that the mild solution stays within a class of bounded variation in this sense of regularity. This shows that this class of functions is an invariant of the semigroup. The same analysis carries over to the periodic problem. The class of bounded L^1-variation functions used here can be normed to give a Banach space structure. It also has an analogue with the space of Lipschitz functions.

MESOSCOPIC RESONANCE OF SELF-EXCITED DEFECT OSCILLATIONS

Lev Steinberg, Mathematics, University of Puerto Rico at Mayaguez

We approach the problem within the framework of continuum mechanics which describes the defective body in terms of dislocation and disclination fields. It is shown that this continuum model produces massless defect-waves under specific condactivity type conditions. It is shown that interactions of the defect waves produce some resonances, which have been experimentaly detected.

HOPF BIFURCATION SURFACES IN PIONEER-CLIMAX COMPETING SPECIES MODELS

Suzanne Sumner, Mathematics Department, Mary Washington College

Pioneer-climax competing species models using differential equations are studied where the per capita growth rates are functions of the total weighted densities of the populations. The per capita growth rate of the pioneer species is monotonically decreasing whereas the per capita growth rate of the climax species is a one-humped function of the total weighted density. Stocking and harvesting of either species are included in the model by constant rate forcing. The Hopf bifurcation surface is found and thus indicates strategies for restoring the system to a stable equilibrium.

EXPERIMENTS WITH A STIFF ODE SOLVER FOR STIFF SYSTEMS OF DELAY DIFFERENTIAL EQUATIONS

Skip Thompson, Mathematics and Statistics, Radford University

In contrast to the state of affairs for nonstiff delay differential equations, there is little high–quality software available for stiff DDE systems. We will discuss the use of a variant, DDRLAG developed initially by D. Sutherland, of David Kahaner's well–known DDRIV ODE solver for the solution of such systems. We will discuss several of the numerical issues which must be considered. We will illustrate the effectiveness of DDRLAG for the solution of three recent models for the spread of infectious diseases.

SYSTEM OF DIFFERENCE EQUATIONS INVOLVED BY THE SIERPINSKI GASKET

Zdzislaw W. Trzaska, Electrical Engineering, Warsaw University of Technology, Warsaw, POLAND
(*joint work with Wieslaw Marszalek*)

The paper aim is focussed on properties and solutions of a system of difference equations resulting from mathematical models of a network with fractal structure like the Sierpinski gasket. The equations system is formed by special form of difference equations in which discrete indeterminates belong to specific sets of integers. Using appropriate methodology solutions are expressed in terms of system parameters and the boundary conditions.

ON SPECTRAL FUNCTIONS OF K.FRIEDRICHS AND M.KREIN BOUNDARY VALUE PROBLEMS

E.Tsekanovskii, Mathematics, Niagara University

We give the complete description of all Stieltjes and Inverse Stieltjes Weyl-Titchmarsh m-functions of minimal positive Schrödinger operator on half-line and establish new properties of spectral functions of K.Friedrichs and M.Krein boundary value problems.The new types of inequalities with strict constants relating to these boundary value problems are obtained as well as connections with some nonselfadjoint boundary value problems.

REGULARITY NEAR THE CHARACTERISTIC SET IN THE NON-LINEAR DIRICHLET PROBLEM AND CONFORMAL GEOMETRY OF SUB-LAPLACIANS

Dimiter Vassilev, Department of Mathematics, Purdue University
(*joint work with Nicola Garofalo, Purdue University*)

We discuss a class of non-linear sub-elliptic problems on stratified nilpotent Lie groups. The classical motivation comes from the Yamabe problem and the problem of determining the best constant in the Sobolev inequality. We are interested in questions of existence and non-existence of weak solutions in various bounded and unbounded domains. New regularity results are obtained at characteristic points of the boundary, under some natuarl geometric conditions. We give examples of unbounded domains for which there is no solution of the considered Dirichlet problem. Finally, in connection with the CR Yamabe problem we exhibit a family of solutions on the whole group.

NUMERICAL METHODS FOR QUASILINEAR SINGULAR PERTURBATION PROBLEMS: SOME RECENT RESULTS

Relja Vulanovic, Mathematics and Computer Science, Kent State University Stark Campus

Several recent results on numerical methods for singularly perturbed quasilinear boundary value problems are surveyed. The problems are of the form

$$-\epsilon u'' - b(x,u)u' + c(x,u) = 0, \quad x \in (0,1), \quad u(0),\, u(1) \text{ given},$$

where ϵ is a small positive parameter. Most of the results treat the case b¿0, but some other cases are also considered. The methods discretize the problem on an improved Shishkin mesh (a piecewise equidistant mesh which guarantees that the numerical solution is uniformly accurate with respect to ϵ). Another improvement is the proof of pointwise accuracy, which is enabled by a special stability inequality.

GLOBAL SOLUTIONS AND STABILITY OF MAGNETOHYDRODYNAMICS

Dehua Wang, Department of Mathematics , University of Pittsburgh

Magnetohydrodynamics (MHD) concerns the motion of conducting fluids in an electromagnetic field with a very wide range of physical applications. There is a complex interaction between the magnetic and fluid dynamic phenomena. Because of this complexity, many major mathematical problems remain open for MHD in any dimension. In this talk, we discuss our research project on MHD, present our results and progress, and outline our on-going studies and open problems. We are interested in the fundamental mathematics problems such as existence, uniqueness, stability, large-time behavior, etc. For the plane magnetohydrodynamic flows, we first establish the existence, uniqueness, and stability of global solutions to the initial-boundary value problem with large initial data in H^1 and show that shock waves do not develop as time evolves for such initial data. For the case of discontinuous initial data, the initial discontinuities propagate as time evolves. The existence of global discontinuous solutions with large discontinuous initial data is obtained. We also analyze the large-time behavior of solutions to certain magnetohydrodynamics.

A DIFFUSION MODEL OF PHENOTYPE EVOLUTION IN *HELICOBACTER PYLORI*

Glenn Webb, Mathematics, Vanderbilt University

Helicobacter pylori are bacteria that colonize the human stomach and are associated with diseases of the upper gastrointestinal tract. About half of the world's population is colonized with *H. pylori*. Because colonization appears to confer benefits (reduced risk of esophageal diseases) as well as costs (increased risk of peptic ulcer disease and gastric adenocarcinoma), it is important to understand the biology of host-microbial interaction. Examination of the selective pressures on *H. pylori* provide a model for evolution of Lewis antigen phenotypes during the colonization of a human host. A nonlinear diffusion equation model of Lewis antigen expression in *H. pylori* will be presented. The model incorporates the following key elements: *H. pylori* strains are highly diverse and continued variation is occuring during colonization; (2) *H. pylori* Lewis expression varies during colonization and host characteristics select for particular phenotypes; and (3) *H. pylori* strains have substantial ability to mutate and to recombine DNA with other *H. pylori* strains, thus providing opportunity for quasi-species development. The model has the form

$$\frac{\partial}{\partial t}u(y,t) = \alpha^2\frac{\partial^2}{\partial y^2}u(y,t) + \Big(\beta(y) - \gamma\int_{y_1}^{y_2} u(\hat{y},t)d\hat{y}\Big)u(y,t) + \tau\Big(H(u(\cdot,t)(y) - u(y,t)\Big) \quad (1)$$

$$\frac{\partial}{\partial y}u(y_1,t) = \frac{\partial}{\partial y}u(y_2,t) = 0,\ t > 0 \quad (2)$$

$$u(y,0) = \phi(y),\ \phi \in L^1_+(y_1,y_2),\ y_1 < y < y_2 \quad (3)$$

where $u(y,t)$ is the density of bacteria with respect to Lewis antigen type y, α^2 corresponds to the rate of mutation, $\beta(y)$ is the selection coefficient, γ is the crowding parameter, and H is the recombination operator. Model simulations will be compared to data.

DEVELOPMENT OF A MULTISTATE BIOLOGICALLY-BASED DOSE-RESPONSE MODEL FOR DEVELOPMENTAL TOXICOLOGY

Shree Y. Whitaker, Mathematics Dept., North Carolina State University
(*joint work with S. Y. Whitaker, C. J. Portier, H. T. Tran, R. E. Chapin*)

The biologically-based dose-response model for developmental toxicology derived by Leroux *et al.* (1996) is extended. The original model had two basic states; precursor cells and differentiated cells with both states subject to a linear birth-death process. Here, a model is developed with a highly controlled birth and death process for the precursor cells which limits the number of replications allowed in the development of a tissue and more closely reflects the presence of a true stem cell population. The mathematical derivation of the Leroux *et al. (1996) model was based upon the Ricatti equations (a series of partial differential equations) which limit further expansion into more realistic models of mammalian development. The same formulae for the probability of a defect (a system of ordinary differential equations) can be derived through the Kolmogorov forward equations due to the nature of this Markov process. This modified approach is easily amenable to the expansion of more complicated models of the developmental process.*

FURTHER ANALYSIS OF THE LIPPMANN-SCHWINGER EQUATION

Christopher Winfield, Mathematics, Lamar University

The Lippmann-Schwinger equation is investigated where the the associated integral operator is not of Fredholm type. We find, using estimates by Friedrichs, that for some potentials the integral operator is compact although not Fredholm. Some weak solutions to the equation are studied in these cases.

A UNIFIED APPROACH TO ROBUST PERFORMANCE OF A CLASS OF TRANSFER FUNCTIONS WITH MULTILINEAR CORRELATED PERTURBATIONS

Julian L. Xu, Math, University of Texas at Brownsville
(*joint work with Lang Wong*)

In this paper, we study the envelope of the Nyquist plots generated by a family of stable transfer function with multilinearly correlated perturbations and show that the outer Nyquist envelope is generated by the Nyquist plots of the vertices of this family. We then apply this result to calculating the maximal H-infinite norm and verifying the strict positive realness condition for uncertain transfer function families. vertex results for robustness analysis are established. We also study the collection of Popov plots of this transfer function family and show that a large portion of its outer boundary comes from the vertices of this family to obtain a strong Kharitonov-like theorem.

LIAPUNOV STABILITY OF SPINNING SPACECRAFT WITH PARTIALLY LIQUID-FILLED CYLINDRICAL TANKS

Julian L. Xu, Math, University of Texas at Brownsville

The liquid-filled system are large-scale system with the distributed parameter and is characterized as nonlinear time-variate coupling systems with infinite degree of freedom. It has been formulated. The stability condition, based on the Liapunov functions are given by the change of energy function E being positive definite. The stability principle for a spinning flexible body is stated as follows: The equilibrium spinning motion of a flexible body is stable if all small displacements of flexible elements tilt the spin axis in such a manner that the combined elastic loads and tilted centrifugal loads tend to decrease the displacement. An Example, a spacecraft with two propellant cylindrical tanks indicates that different approaches and approximations used to calculate the change in the moment of inertia due to the liquid propellant motion have different stability conditions.

ON THE CRITICAL EXPONENT FOR A DISSIPATIVE SEMILINEAR WAVE EQUATION

Borislav Yordanov, Mathematical Sciences, UW-Milwaukee
(*joint work with Grozdena Todorova*)

We study the global existence and blowup of solutions of the equation $\Box u + u_t = |u|^p$ in R^n. We show that the exponent $p_c = 1 + 2/n$ is critical for this problem. More precisely, if $p > p_c$, then all solutions with small initial values exist globally. If $p < p_c$, then all solutions with positive initial values blow up in finite time.

TBA

Yuncheng You, Mathematics, University of South Florida

TBA

CONSTRUCTION OF LIAPUNOV FUNCTIONALS FOR LINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS AND STABILITY OF DELAY SYSTEMS

Bo Zhang, Department of Mathematics and Computer Science, Fayetteville State University

Inspired by the construction of Liapunov functionals for linear Volterra integrodifferential equations, we prove some general stability theorem for functional differential equations with infinite delay and weaken the usual requirement for positive definiteness of Liapunov functionals involved in stability theory.