In this notebook, you will learn how to use what you learned in Excel Lab 3 to perform regression for some nonlinear models. Open a new Excel workbook, and perform the steps in this tutorial as we go.

**Regression with Power Functions:**

In class, we learned that finding a power function $y = Ax^p$ by regression on a data set consists of the following steps:

1. Transform the data using a log-log transform (i.e. taking the natural log of both data columns).
2. Use linear regression to find the best-fit line for the transformed data.
3. Use the slope and intercept from the best-fit line to compute values for the constants $A$ and $p$ in the power function.

(NOTE: Fitting an exponential function $y = Ae^{rt}$ is done in an analogous way – see below.)

Since we already learned (in Excel Lab 3) how to do Step 2, all we really need to learn here is how to do Steps 1 and 3 in Excel. Study the following example:

**Example:**

*(Fitting a power function model to a data set.)* We want to fit a power function to this data set:

<table>
<thead>
<tr>
<th>t</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7.0</td>
</tr>
<tr>
<td>3</td>
<td>30.1</td>
</tr>
<tr>
<td>4</td>
<td>77.4</td>
</tr>
<tr>
<td>5</td>
<td>160.6</td>
</tr>
<tr>
<td>6</td>
<td>300.0</td>
</tr>
</tbody>
</table>

To perform Step 1 above, we enter our data into Excel, then create two new columns (one for ln(t) and the other for ln(B)). To compute the values for the ln(t) column, we simply enter it like any other formula, using Excel’s name for the natural logarithm function, LN( ). For example, in the spreadsheet shown below, we enter the formula $=\text{LN}(C4)$ into cell E4.
After entering this formula, use the mouse and cursor to drag it down to the other cells in the column. Repeat this for the ln(B) column, with the obvious changes. Our data table now looks like this:

Step 1 is now complete. Step 2 is just fitting a regression line to the transformed data. We already know how to do this, thanks to Excel Lab 3. We simply compute the slope and intercept of this line, using the transformed data, and arrive at the following:
Step 2 is now complete. For Step 3, we just need to remember how the power function model and the transformed linear model are related. To figure this out (which does not require *Excel*), start with our power function model:

$$B = At^p,$$

where $A$ and $p$ are the constants to be determined. We begin by taking the natural log of both sides:

$$\ln(B) = \ln(At^p).$$

Using properties of logarithms, we can do this:

$$\ln(B) = \ln(A) + p\ln(t).$$

This gives a linear relation between $\ln(t)$ and $\ln(B)$, with the slope given by $p$, and the intercept given by $\ln(A)$. Comparing this to the regression line equation from this example:

$$\ln(B) = m\ln(t) + b,$$

where $m$ is the slope and $b$ is the intercept, we have the equations

$$p = m$$

and

$$\ln(A) = b.$$ 

So, once we have the regression line slope $m$ and intercept $b$ for the transformed data, the corresponding power function model for the original data is given by

$$B = At^p,$$

where
\[ p = m \text{ and } A = e^b. \]

Now, back to Excel to compute \( p \) and \( A \) (labeled “power” and “constant” in the spreadsheet below). For the power \( p \), we just use the address for the slope \( m \):

\[ =E11 \]

For the constant \( A \), we use Excel’s name for the base-e exponential function to enter the formula:

\[ =\text{EXP}(E12) \]

Here’s what we get:

So, our power function model in this case is, with values rounded off,

\[ B=0.68t^{3.4}. \]

So, Step 3 is done. The last thing to do is to check if our model is sensible, by computing the values of this model at the \( t \)-values in the data table, and by plotting the model and the data together. We already learned how to do this in Excel Lab 3, in the case of a linear model. The only difference here is in the formula that we use to compute the column of model values. In this case, we are entering the power function formula, so that, for example, the formula that goes in cell G4 is

\[ =E$14*C4^E$13. \]

(Note the use of the $ in the appropriate places.)
Now, make a scatter plot of the original data set, and add the power function values as a smooth curve to get this:

The data points are not perfectly on the curve, but pretty close.

**Regression with Exponential Functions:**

To fit an exponential function $y = Ae^{rt}$ to a data set, we do the same things as we do for a power function fit, with the following changes:

- When transforming the data, we only take the logarithm of the dependent variable values ($B$). We use $t$ in place of $\ln(t)$ in the stuff above.
- You have to start with the exponential model formula, rather than the linear one, to compute the relationships between the slope and intercept of the regression line, and the constants $A$ and $r$ from the exponential model equation.