Excel Lab 4: Estimating Area Under a Curve

In this lab, we use Excel to compute $L_n$, $R_n$, $M_n$, and $T_n$ for different values of $n$, given a function $f(x)$ and an interval $[a,b]$.

Example:

Estimate the area under the curve $f(x) = xe^x$ for $x$ between 2 and 5 using 10 subintervals.

First we create two columns that give us the left and right endpoints of each of the 10 subintervals on $[0,5]$ that we need:

Label the columns, Left Endpoint, Right Endpoint, Midpoint, $L,R$, $M$, and $T$. In the Left Endpoint column, in the first box (A2) we enter 2 (the left endpoint of the first subinterval).

In the next box (A3) we enter “A2+ size of subinterval” or in this case: =A2+3/10. We can then drag down 8 more boxes to get the left endpoint of all 10 subintervals.
Similarly, we complete the Right Endpoint column: the first entry (B2) should be the left endpoint of the second interval. We enter =A3 in B2.

Then in B3 we enter =B2+3/10. We can then drag down 8 more boxes to get the right endpoint of each 10 subintervals. The right endpoint of the last subinterval should be the original right endpoint (5 in this case).

Next we compute the midpoint of each subinterval using the average of the left and right endpoints:
You can drag down this formula to get the other midpoints:

It is a good idea to check at this point that what you have makes sense as left endpoint, right endpoint, and midpoint of each of the 10 subintervals.
Now we simply enter the correct formula for the area of the rectangle for
the first subinterval in the first row for each of $L_n$, $R_n$, and $M_n$. For $T_n$ we
use the average of $L_n$ and $R_n$. In this case, under $L_n$ we have
$=\frac{3}{10}A2*\text{EXP}(A2)$.
For $R_n$ we have $=\frac{3}{10}B2*\text{EXP}(B2)$. And for $M_n$ we have
$=\frac{3}{10}C2*\text{EXP}(C2)$. For $T_n$ we simply use $=(D2+E2)/2$.

We can drag down each column to complete.

The final estimate is the sum of each of the columns $L_n$, $R_n$, $M_n$ and $T_n$.
You can highlight each column and hit the summation button on the toolbar:

So the final answer is $L_n = 483.7, \ R_n = 701.9, \ T_n = 592.8, \ and \ M_n = 583.0$
Finally, to change $n$, we simply need to delete the sum row and change the “3/10” in boxes A3, B3, D2, E2, and M2 to $3/(\text{new } n)$. Then we can drag down each column.

**Exercises.**

1. **Compute** $L_n$, $R_n$, $T_n$, and $M_n$ using $n = 2, 10, 50, \text{ and } 250$ to estimate the area under the graph of $f(x) = \frac{1}{x}$ for $x$ between 1 and 2. Give 8 decimal places.

   Show (print) your calculations for $n=2$ and $n=10$.

2. We know the correct area in this particular case: $\ln(2)$. Therefore, we compute the error in each estimate. For example, error in using $L_n$ is computed as $L_n$ minus $\ln(2)$.

   Complete the table below. Give 8 decimal places.

<table>
<thead>
<tr>
<th>$n$</th>
<th>Error using $L_n$</th>
<th>Error using $R_n$</th>
<th>Error using $T_n$</th>
<th>Error using $M_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. For each method, fill in the blank: Increasing $n$ by a factor of 5 decreases the error using [put in each method] by a factor of approximately ____________.  
   (Hint: look at ratios in the table you made in (2).)